The Value of Research: Skill, Capacity, and Opportunity

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"When you can measure what you are speaking about, and express it in numbers, you know something about it."

Lord Kelvin, 1883

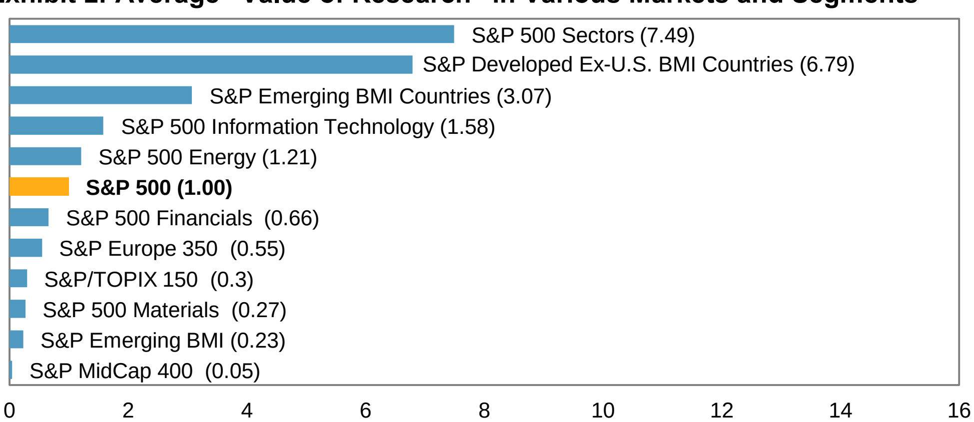
EXECUTIVE SUMMARY

How much should a portfolio manager be willing to pay for research? The question is of importance to *any* manager, but has become particularly pertinent since newly imposed European rules require that the costs of investment research—previously offered by many investment banks as an in-kind consideration in return for brokerage business—be unbundled from trading.

Unfortunately, attempts to determine a fair value for research in the most general circumstances are doomed to fail. Even if we only consider direct recommendations to buy or sell certain securities, the value of such recommendations to a portfolio manager will vary according to the absolute size of positions taken in response. **Instead, we provide a framework for estimating *relative* research values across markets and constituents, under certain stylized (but reasonable) assumptions**.

Exhibit 1 provides a summary of our main result—comparing the putative value of recommendations in selected markets, expressed as a multiple of the equivalent measure applied to stock-based recommendations within the S&P 500®.

Exhibit 1: Average "Value of Research" in Various Markets and Segments



Source: S&P Dow Jones Indices LLC. Data based on monthly averages from October 2008 to September 2018. Chart is provided for illustrative purposes. Past performance is no guarantee of future results.

INTRODUCTION: THE IMPACT OF "UNBUNDLING" RESEARCH COSTS

The relevant regulatory change made by the MiFID II…

The Markets in Financial Instruments Directive (MiFID) II is an updated version of a regulation that has been in force throughout the European Union (EU) since November 2007.[1] The update came into effect on January 3, 2018, and seeks "to reform market structures, bring more transparency to the trading of financial instruments, and strengthen investor protection."[2]

…is that execution costs and charges must be separated, or "unbundled," from the cost of research…

For our purposes, the relevant regulatory change is that execution costs and charges must be separated, or "unbundled," from the cost of research, and that investment managers must either absorb research costs or explicitly pass them on to their clients under pre-agreed terms.[3] Since investment managers were formerly allowed to pay for research by the allocation of client trading commissions, MiFID II has the potential to produce major changes in the economics of research sales.

While these rules are of most immediate concern to institutions operating in the EU, **MiFID II has potential global implications**: the updated directive applies to *all* firms that conduct business in Europe, and many expect the legislation to be extended to other regions.[4,5]

…and that investment managers must either absorb research costs or explicitly pass them on to their clients under pre-agreed terms.

From a practical perspective, MiFID II requires managers to set research budgets and to decide where to spend them. Obviously, the size of a particular research budget will depend on idiosyncratic factors, such as a firm's assets under management. But when it comes to allocating resources**, the *relative* value of research is likely to be comparable**—if I find one analyst's recommendations to be worth double those of other analysts, it is reasonable to hypothesize that these recommendations would also prove to be twice as valuable to anyone else.

This paper argues that the relative value of research is driven by a combination of three things: the **information content** of the research, the **dispersion** within the market where recommendations are made and implemented, and the **capacity** of each market to allow for active positions of varying sizes. While we do not claim to offer a universally applicable framework for setting research budgets, **we hope to offer a practical and useful way to think about the value of signals** for markets of varying size, concentration, and risk levels.

[1] For more details on the original directive, as well as the updated form, please see the European Securities and Markets Authority (ESMA) webpage.

[2] Preece, Rhodri, "MiFID II: A New Paradigm for Investment Research," *CFA Institute*, 2017.

[3] Ibid.

[4] Moore, Howard, "Seeing the Market More Clearly," *Institutional Investor*, June 14, 2018.

[5] "MiFID II is driving global standards for research unbundling, says RSRCHXchange," *Institutional Asset Manager*, June 12, 2018.

THE UNKNOWN VARIABLE OF "INFORMATION CONTENT"

Some research reports provide explicit recommendations and quantitative predictions—"buy," "sell," or "hold," price targets, future earnings estimates, and so on. Other reports offer more implicit or conditional recommendations, which may be accompanied by an associated degree of conviction. The success or failure of such recommendations can be measured in a multitude of ways, the simplest of which is a "hit rate" (the percentage of times that following a set of recommendations would have improved performance).

The term "information coefficient" typically represents a correlation between predictions and outcomes.

More generally, **the term "information coefficient" (IC) typically represents a correlation between predictions and outcomes**.[6] For example, we might measure the correlation between future percentage changes implied by a series of price targets offered at the start of a period to the actual percentage changes in price measured at the end of the period.

Measures of skill tend to be historically biased and hard to measure at the benchmark level.

It would be convenient if one could ascertain the IC of research in advance. However, **measures of skill tend to be historically biased and hard to measure at the benchmark level**—except to the extent that S&P Indices Versus Active (SPIVA®) reports demonstrate the difficulty of maintaining a high batting average.[7] To avoid paralysis by such uncertainty, for the following sections, **we shall assume that the IC is a small positive number that is known in advance by the manager**.[8]

We assume that the IC is a small positive number known in advance by the manager.

If the IC is a given, the next important factor is the magnitude of excess performance that predictive recommendations can generate.

DISPERSION AND MEASURING OPPORTUNITY FOR SKILL

Suppose one could purchase a subscription to signals recommending purchases or sales of securities in two different markets (or time periods), both with a similar degree of accuracy. In which market should a recommendation be preferred?

[6] The terms "information content" and "information coefficient" are sometimes used interchangeably. We shall use the former in the general sense of accurate predictions, reserving the latter term for the case where such accuracy is measured by a correlation statistic.

[7] SPIVA Scorecards illustrate the relative rarity with which active funds outperform their benchmarks and can be found at https://spindices.com/SPIVA.

[8] We may as well assume the IC is positive, since a negative IC, if known in advance, offers a contrarian indicator.

A report with perfect foresight will be of little value if the difference in performance between the relative winners and losers is minimal.

Clearly, even a report with perfect foresight will be of little value if the difference in performance between the relative winners and losers is minimal. We must therefore consider **the dispersion of the market in which the recommendation is being made**.[9]

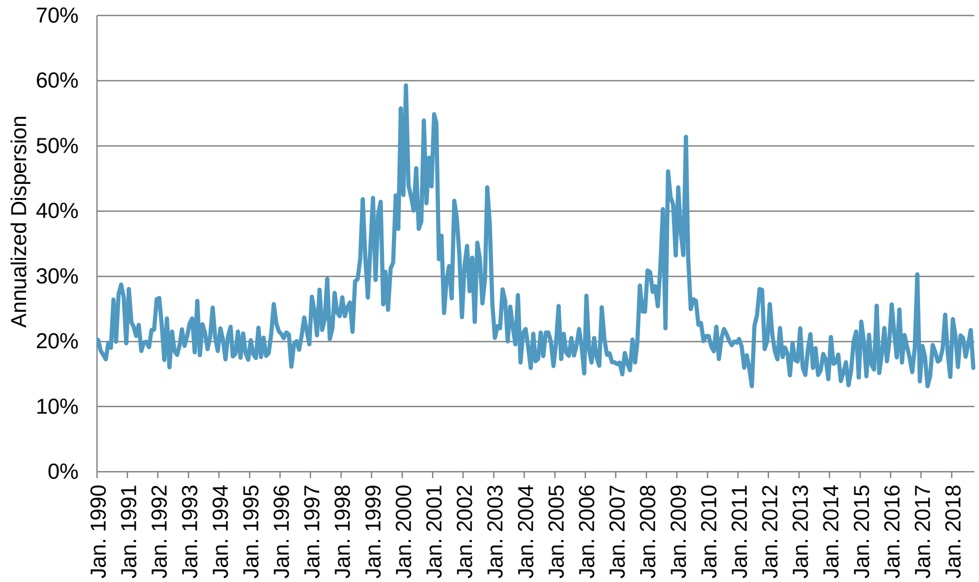
We must consider the dispersion of the market in which the recommendation is being made.

In order to illustrate the time-varying levels of dispersion, Exhibit 2 shows monthly dispersion levels in the S&P 500 since January 1990—calculated as weighted, annualized standard deviations among the monthly returns of S&P 500 constituents. Absent any other changes, *correct* recommendations to buy or sell constituents would have been better rewarded in high-dispersion periods, such as 1999 or 2000, than in less-disperse periods, such as 2013 or 2014.

Correct recommendations to buy or sell constituents would have been better rewarded in high-dispersion periods…

…compared with low-dispersion periods.

Exhibit 2: S&P 500 Dispersion Peaked in 1999-2001



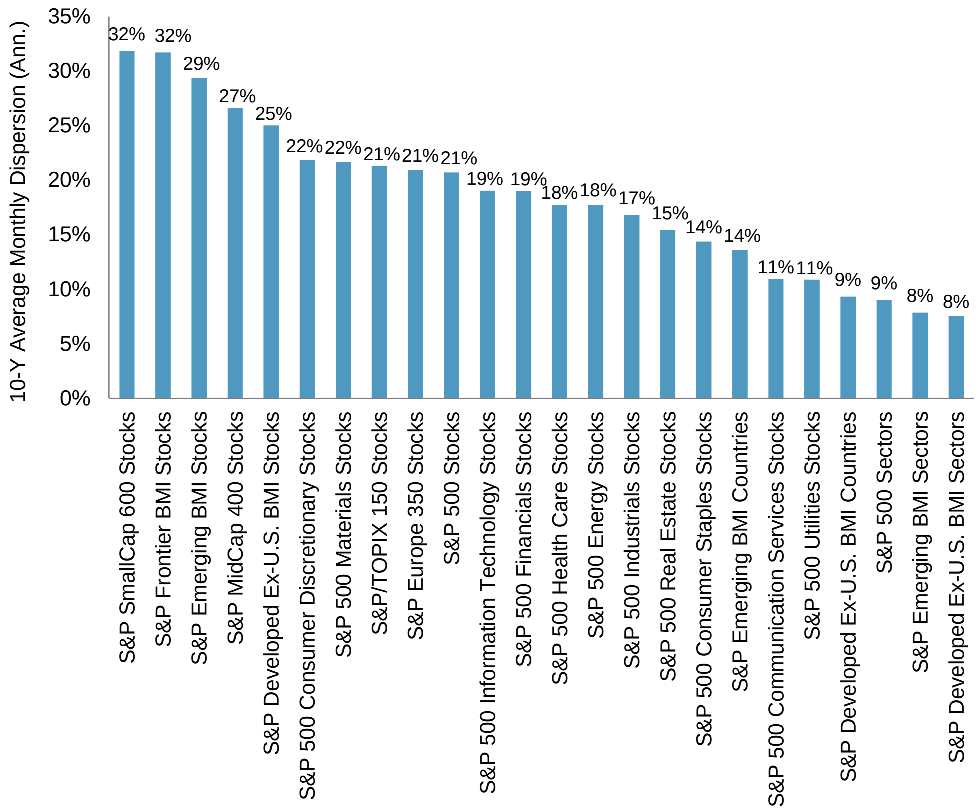
Source: S&P Dow Jones Indices LLC. Based on annualized monthly data from January 1990 to September 2018. Chart is provided for illustrative purposes. Past performance is no guarantee of future results.

Offering a cross-market perspective, Exhibit 3 shows average (monthly annualized) dispersion figures for a variety of markets over the 10-year period ending in September 2018. Note that we allow for markets to be divided with different levels of granularity. For example, we can measure the dispersion among countries and sectors, as well as among stocks, for indices such as the S&P Emerging BMI.

9 For more details on how dispersion can help to measure market opportunity, please see Edwards, Tim and Craig J. Lazzara, "Dispersion: Measuring Market Opportunity," *S&P Dow Jones Indices LLC*, December 2013.

We can measure the dispersion among countries and sectors, as well as among stocks.

Exhibit 3: Average Dispersion of Various Markets and Segments



Source: S&P Dow Jones Indices LLC. Data based on average annualized monthly dispersion figures from October 2008 to September 2018. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance. Past performance is no guarantee of future results.

A basket of the better-performing stocks in the S&P Frontier BMI would likely have delivered considerably higher outperformance…

…than a basket of the better-performing single-sector indices selected from the S&P Developed Ex-U.S. BMI.

Exhibit 3 provides a measure of outperformance opportunity. A basket of the better-performing stocks in the S&P Frontier BMI would likely have delivered considerably higher outperformance than a basket of the better-performing single-sector indices selected from the S&P Developed Ex-U.S. BMI, for example. Similarly, the returns from making successful bets among U.S. small caps, as represented by the S&P SmallCap 600®, would appear to be higher than making similarly successful bets among S&P 500 sectors; the dispersion among the former was more than three times higher.

For market participants of sufficiently small size, dispersion alone may be an appropriate guide to the value of research.

For market participants of sufficiently small size, dispersion alone may be an appropriate guide to the value of research. Such an investor could form a view of the relative value of research on various markets simply by rescaling the data of Exhibit 3. For example, recommendations pertaining to S&P SmallCap 600 members would be over 50% more valuable than equally accurate recommendations for S&P 500 constituents. Recommendations of S&P 500 sectors would be worth less than one-half the value of equally accurate recommendations of S&P 500 stocks.

However, research is not automatically worth more when dispersion is higher.

For most investors and asset owners, however, **research is not automatically worth more when dispersion is higher, due to the problem of varying capacities**. If an investor has a single U.S. dollar to deploy, the potential outperformance from picking wisely among S&P SmallCap 600 stocks might be higher than picking among S&P 500 sectors, but if managing a multi-billion-dollar portfolio, the investor will be more limited in the extent to which active positions in small-cap stocks can be established.

Many readers will be familiar with the so-called "Fundamental Law of Active Management."

We can adjust dispersion to incorporate these capacity issues, and the degree of portfolio "active share" they imply, by placing both in the context of an existing measure of the value of skill to active managers—the so-called "Fundamental Law of Active Management."

DISPERSION AND THE FUNDAMENTAL LAW OF ACTIVE MANAGEMENT

Many readers will be familiar with the Fundamental Law of Active Management, first outlined in the late 1980s.[10] It states that an active manager's information ratio (IR) is positively related to his skill in identifying opportunities for excess returns (IC), scaled by the number of independent bets that manager has implemented (breadth or BR). The functional form is given by the following equation:

The law offers a guiding principle for interpreting the impact of skill on risk-adjusted active returns.

$$\boldsymbol{IR = \sqrt{BR} \times IC}$$

As a heuristic, the equation offers a guiding principle for interpreting the impact of skill (in our case, the IC of research reports) on risk-adjusted active returns. The Fundamental Law offers the considerable appeal of simplicity and provides a wholly intuitive interpretation of the investment management process.[11]

A simplified example can illustrate the potential roles of **IC, dispersion, concentration, and active share** in determining the return and information ratio of active portfolios.[12] We assume that excess returns are random and identically distributed across all stocks, and that an analyst's recommendation to buy or sell is implemented by overweighting or underweighting at the limit of some predetermined capacity in each position.

After these simplifications, we may write the expected active return and information ratio of a portfolio implementing recommendations as follows:

[10] Grinold, Richard C., "The fundamental law of active management," *The Journal of Portfolio Management,* Spring 1989.

[11] In what might be a potentially flattering comparison between the speed of light and the skill of active managers, if skill is represented as c and breadth by M, we can square both sides and define some E, a term related to the value added by a manager, which is equal to Mc^2. This formulation was part of Grinold's original explanation of the law, and may account for some of its subsequent fame.

[12] See Appendix A for full details of our simplification, along with a limited amount of algebra.

$$\textbf{\textit{Expected Active Return}} = \boldsymbol{IC} \times \boldsymbol{D} \times \boldsymbol{TAS}$$

$$\textbf{\textit{Expected Information Ratio}} = \boldsymbol{IC} \times \sqrt{\boldsymbol{M}} \times \boldsymbol{TAS}$$

Where

- $\boldsymbol{IC}$ is the information coefficient, defined as the correlation between the predicted sign of excess returns and the realized excess returns in recommended constituents,
- $\boldsymbol{D}$ is the capacity-weighted dispersion of the constituents,
- $\boldsymbol{TAS} = \sum w_i$ is the total active share, which is equal to the sum of the absolute values of the active weights w_i ,
- $\boldsymbol{M} = \left(1/\sum w_i^2\right)$ is a measure of the "effective number" of positions in the active portfolio, which is equal to the reciprocal of the sum of the squared active weights.[13]

The active return equation offers more than a formalization of obvious truths.

It identifies the relative role of skill, active share, and dispersion in active returns.

It is worth emphasizing that **the active return equation offers more than a formalization of obvious truths; it identifies the *relative* role of skill, active share, and dispersion in active returns**. A 10% increase in any one of IC, active share, or dispersion would precisely offset a 10% decline in one of the others. As a base for exploration of the value of signals to portfolios, these equations offer a promising start and an intuitive interpretation.

INTEGRATING RESEARCH COSTS

How much should a manager be willing to pay to receive recommendations to overweight or underweight? The addition of fixed research costs makes no difference to return volatility in our simple example,[14] but the cost per recommendation (C), expressed as a percentage of portfolio value, must be justified by an incremental expected return.

Larger absolute research costs are justifiable when their information content is higher, in markets where there is higher dispersion, or for active strategies with higher total active share, or for larger portfolios.

In our model of simple, directional recommendations to overweight or underweight, the expense in obtaining recommendations would be justified if it satisfies the following equation.

$$C < IC \times D \times \frac{TAS}{N}$$

Here N is the number of research recommendations required to determine the active positions. **The equation reflects the real world observation that, all else being equal, larger absolute research costs are justifiable**

[13] We prefer TAS or total active share, since convention limits the term "active share" to the sum of the absolute values of the active weights *divided by 2*. See Appendix B for further remarks on the "effective number," M, and its relation to measures of concentration.

[14] By assuming that excess returns are independent and identically distributed, we avoid the much more difficult problem of portfolio construction where returns might be correlated.

when their information content is higher, in markets where there is higher dispersion, for active strategies with higher total active share, or (since C is expressed as a percentage of the notional portfolio size) for larger portfolios.

INCORPORATING CAPACITY

In frontier markets, the expected percentage reward from successful active bets might be greater…

Exhibit 3 showed that there was typically much higher dispersion in the monthly performance of the S&P Frontier BMI's constituents than among—for example—those of the S&P 500. However, frontier market stocks are on average *much* smaller than S&P 500 members; the former had an (index-weighted) average capitalization of $2.7 billion versus $218.9 billion for S&P 500 constituents (a ratio of more than 80 to 1).[15] **In frontier markets, the expected *percentage reward* from successful active bets might be greater, but the *size of an investor's position* might have to be smaller**.

…but the size of an investor's position might have to be smaller.

How should we account for capacity in our analysis?

How should we account for capacity in our analysis? It's an elusive concept, not least because to a large degree, it depends on each investor's wealth and constraints. A $100 million hedge fund, in other words, would have fewer capacity constraints than a $100 billion sovereign wealth fund. Despite the individuality of investors' capacity constraints, there may be a common, or generally applicable, element, and that element is market capitalization.[16] There is, in other words, more capacity to maintain overweights or underweights in Apple, the largest stock in the S&P 500, with its $1 trillion market capitalization, than there is to do the same in Quanta Services, currently the smallest stock in the S&P 500, with a capitalization of $5 billion.

We might reasonably suppose that an investor's ability to take active positions in each index constituent is proportional to that constituent's market capitalization.

We might reasonably suppose that **an investor's ability to take active positions in each index constituent is proportional to that constituent's market capitalization**. This assumption does not require the active portfolio to be particularly large to be realistic: real-world variables such as trade impact costs and position limits often scale proportionally to market capitalization, at least in approximate terms.[17] The assumption allows us to examine the potential value of recommendations in various markets and market segments as a function of the dispersion and average capitalization of an index's constituents.

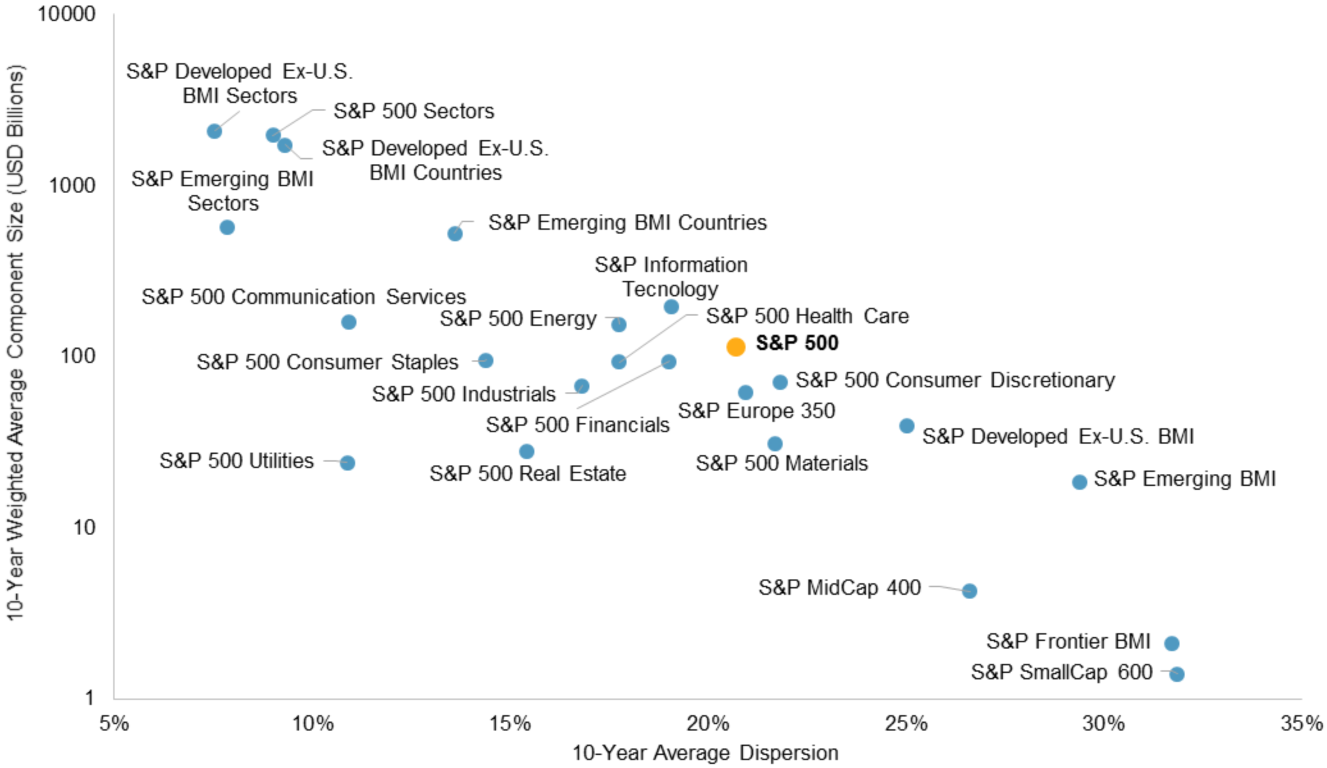
[15] Constituent size data in this section reflect free-float market capitalizations as of Sept. 30, 2018. Source: S&P Dow Jones Indices LLC.

[16] To be precise, float-adjusted market capitalization.

[17] Importantly, a long-only manager's ability to take negative bets is completely constrained by the weight of each stock in his benchmark.

Exhibit 4 plots a comparison between the average monthly dispersion and the average capitalization of constituents (or segments) for the same set of markets as Exhibit 3. Note also that the vertical axis of average constituent capitalization is in logarithmic scale (and that dispersion and capitalization data represent 10-year averages). We suppress the segment name if the calculation is based on the underlying stock-level data (hence "S&P 500" instead of "S&P 500 stocks").

Exhibit 4: An Inverse Relationship Between Constituent Size and Dispersion



Source: S&P Dow Jones Indices LLC. Data based on annualized monthly dispersion and capitalization figures from October 2008 to September 2018. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance. Past performance is no guarantee of future results.

Generally speaking, the negative relationship plotted in Exhibit 4 demonstrates a broader truth: **higher dispersion frequently comes with a trade-off against greater granularity in constituents**.

MEASURING RELATIVE RESEARCH VALUE: CAPACITY-ADJUSTED DISPERSION

If we suppose that active positions are scaled in proportion to available capitalization, then all else being equal, **recommendations pertaining to a market segment with double the dispersion are just as valuable as equally accurate recommendations pertaining to a market segment with double the capitalization**.

In order to facilitate direct comparisons, we define a benchmark's **capacity-adjusted dispersion (*CAD*)** as the product of the dispersion of the index's constituents and their average size.

$$CAD = Dispersion \times Average\ Constituent\ Size$$

In Exhibit 4, the farther up and to the right that an index plots, the greater its capacity-adjusted dispersion.

In order to facilitate direct comparisons, we define a benchmark's capacity-adjusted dispersion as the dispersion of the index's constituents times their average size.

The concept of capacity-adjusted dispersion has intuitive appeal; it might be interpreted as the potential scale of the available bets (capitalization) multiplied by the potential magnitude of the returns to successful bets (dispersion).

This formulation allows us to examine a given benchmark at different degrees of granularity.

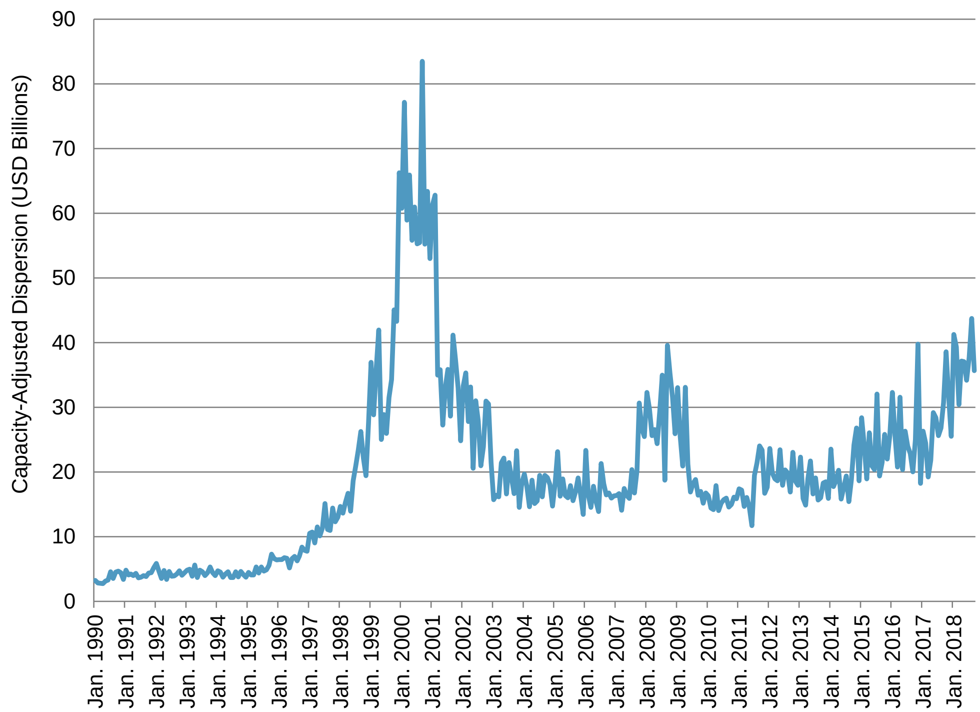
This formulation allows us to examine a given benchmark at different degrees of granularity. Instead of analyzing constituent stocks, we can group those stocks into segments such as sectors or countries. Of course, if we measure the dispersion of single-country returns within a global benchmark, we also need to measure the average capitalization at the single-country level to compute the capacity-adjusted dispersion.

Capacity-adjusted dispersion also has a particular advantage—it is quite simple to observe empirically. Benchmark weights, capitalizations, and constituent returns are all that is required in order to calculate the capacity-adjusted dispersion in each period. Exhibit 5 shows the evolution of the capacity-adjusted dispersion for the S&P 500.

Exhibit 5 shows how the increased concentration of capital into the largest names of the S&P 500 during the late 1990s technology bubble…

…accentuated the degree of active opportunity.

Exhibit 5: Monthly Capacity-Adjusted Dispersion of the S&P 500



Source: S&P Dow Jones Indices LLC. Data based on annualized monthly dispersion and capitalization figures from January 1990 and September. 2018. Chart is provided for illustrative purposes. Past performance is no guarantee of future results.

It is illustrative to compare Exhibit 5 with simple S&P 500 dispersion as illustrated in Exhibit 2. While the peak in dispersion during the technology bubble was already clear from Exhibit 2, Exhibit 5 shows how **the increased concentration of capital into the largest names of the S&P 500 during the late 1990s accentuated the degree of active opportunity**. More recently, the increases in concentration and capitalization observed in the S&P 500 during 2018 have produced a striking increase in capacity-adjusted dispersion, at a time when dispersion itself has increased only moderately. This implies an increased *absolute* capacity for overweighting or underweighting active positions and a greater opportunity for active outperformance (or embarrassment) in absolute terms.

RELATIVE MEASURES OF CAPACITY-ADJUSTED DISPERSION

While absolute measures are useful, it seems more natural to express the value of recommendations in relative terms.

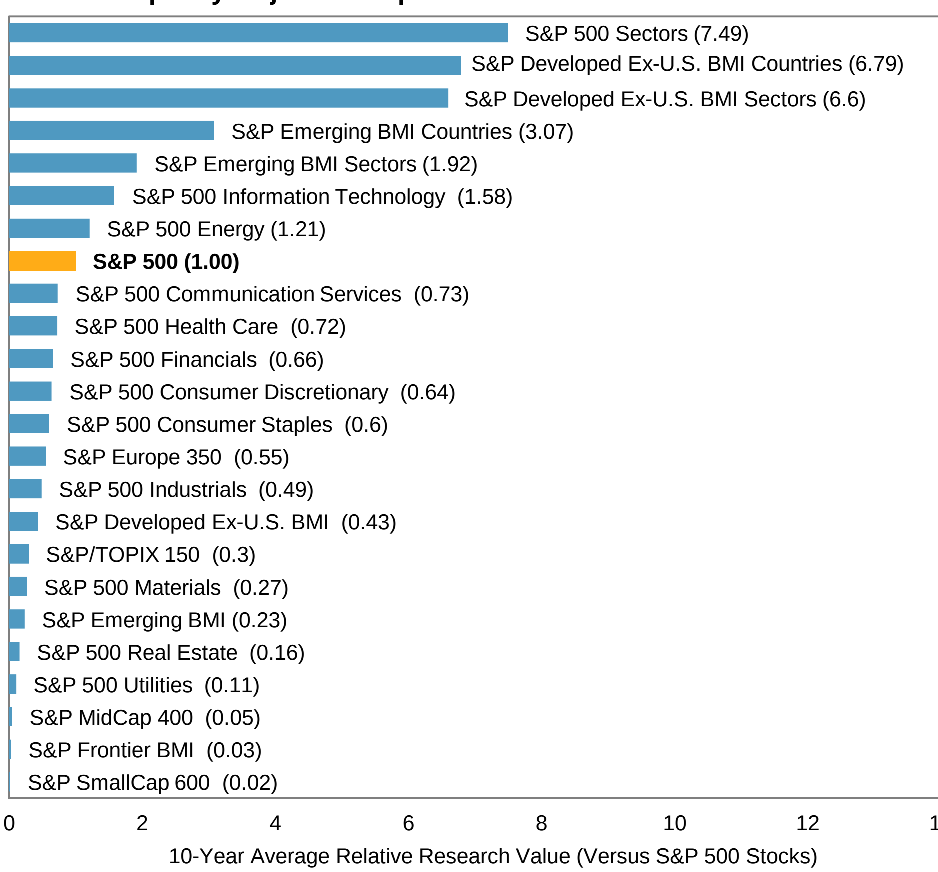
While absolute measures are useful, it seems more natural to express the value of recommendations in relative terms. For example, what value might one place on advice to buy or sell certain S&P 500 *sectors* in comparison to S&P 500 *stocks*?

Taking the capacity-adjusted dispersion of S&P 500 stocks at any particular point in time as the fundamental unit of comparison…

…Exhibit 6 displays the 10-year average capacity-adjusted dispersion of various markets expressed as a multiple of the equivalent measure applied to S&P 500 stocks.

Taking the capacity-adjusted dispersion of S&P 500 stocks as the fundamental unit of comparison, Exhibit 6 displays the ratio between the 10-year average capacity-adjusted dispersion of various markets compared to that of S&P 500 stocks.

Exhibit 6: Capacity-Adjusted Dispersion Across Various Markets



Source: S&P Dow Jones Indices LLC. Data based on annualized monthly dispersion and capitalization figures from October 2008 to September 2018. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance. Past performance is no guarantee of future results.

Accordingly, Exhibit 6 offers a quantitative basis for direct comparisons. We might say, for example, that the most valuable research to a manager benchmarked against the S&P Developed Ex-U.S. BMI would be to know which countries to overweight or underweight, while research with a similar IC on S&P 500 sectors could be worth more than seven times as much as research on S&P 500 stocks.

The idea that research at the sector or country level is the most valuable, followed by stocks, is a powerful finding and supports a top-down approach to asset allocation.

The idea that **research at the sector or country level is more valuable, followed by research on stock selection, is a powerful finding and supports a top-down approach to asset allocation**. This has significant implications for the asset management industry, with potential for less reliance on sell-side analysts' coverage of individual stocks.

Exhibit 6 also provides a way to investigate what level of information coefficient would justify a similar cost for recommendations in two markets of different size or risk levels.

Additionally, Exhibit 6 allows us to determine the relative difference in IC required to make the value of research in one market equivalent to the value of research in another. For example, the IC of reports covering S&P Europe 350 stocks would have needed to be 1.8 times larger, on average, than that covering S&P 500 stocks to make research coverage on the former as valuable as those of the latter, *ceteris paribus*.

In passing, we note that the scale of differences in Exhibit 6 allows for the possibility that research conducted in more granular segments such as small-cap or frontier markets must have considerably higher predictive power to be attractive. This offers some theoretical support to the widespread (and somewhat intuitive) assumption that research conducted at a more granular level, and on less-popular segments of the market, might offer the potential for a higher success rate.

CONCLUSION

Capacity-adjusted dispersion measures may assist managers in determining the optimal markets or segments for the expression of active positions.

The most fundamental aspect in valuing research is the accuracy (or predictive power) of the quantitative or qualitative signals supplied. However, while the information content of research reports might be estimable ex-post, it is hard to know in advance. Investors and intermediaries considering the cost of receiving research must instead form a view on the likely performance of the associated predictions. Once they have done so, the notion of capacity-adjusted dispersion may assist in deciding where **the rewards for outperformance, and the capacity to capture them**, combine to offer the best market or segments for the expression of active positions.

APPENDIX A: A STYLISTIC EXAMPLE OF DISPERSION'S ROLE IN ACTIVE RETURNS

In order to construct a simplified and stylistic example,[18] suppose for a single time period there is an independently and identically distributed collection of zero-mean random excess returns S_i with standard deviation $D.$ In notational terms:

> S_i are single period excess constituent returns for $i = 1, 2, ...$
>
> with expectation and volatility $E(S_i) = 0$ and $\sigma(S_i) = D$, respectively.

Suppose we are provided with near-random digital buy or sell recommendations ($R_i = \pm 1$), equal to plus or minus one, that display a correlation (IC) to the excess return of the constituents, but are otherwise noisy.

> $R_i = 1$ or $R_i = -1$ are buy or sell recommendations for $i = 1, 2, ...$; and
>
> $IC = Corr(R_i, S_i)$ is the correlation between signal and ex-post returns.

Suppose also that recommendations to buy or sell are equally likely, and that the (unweighted) returns from taking recommendations were independent.

> $E(R_i) = 0$; and
>
> $Corr\left(R_i S_i, R_j S_j\right) = 0$ for any different pair of distinct recommendations $i \neq j$.

Completing the setup, suppose the active weights are—in absolute terms—determined in advance according to some "capacity," and there are exogenous active weights w_i such that:

> $w_i =$ active weight in the i^{th} recommended security with associated active position equal to $w_i R_i$.

We also have the total active share (TAS) and effective number of active positions (M),

> $TAS = \sum w_i$ and $M = \left(\frac{1}{\sum w_i^2}\right)$.

Note that if the R_i are a series of 1s and -1s with equal probability of either, there is then only one possible distribution overall for the R_I: binomial with unit variance.

Then, in our simplification, the expected excess return $E(P)$ of the active portfolio is given by:

$$E(P) = E\left(\sum w_i R_i S_i\right) = E(R_i S_i) \times \sum w_i$$

Recalling that $Corr(X, Y)\sigma(X)\sigma(Y) = Cov(X, Y) = E(XY) - E(X)E(Y)$, we may substitute R_i for X and S_i for Y, along with our earlier definitions and conditions to obtain:

[18] The role of dispersion in the Fundamental Law of Active Management is considered for more general portfolios (if not all portfolios) in Gorman, Larry R., Steven G. Sapra, and Robert A. Weigand, "The role of cross-sectional dispersion in active portfolio management," *Investment Management and Financial Innovations,* Vol. 7, Issue 3, 2010.

$$E(R_i S_i) = IC \times D$$

And so

$$E(P) = IC \times TAS \times D.$$

Meanwhile, and recalling the definition of M, the variance $Var(P)$ of the excess returns of the active portfolio is given by:

$$Var(P) = Var\left(\sum w_i R_i S_i\right) = Var(R_i S_i) \times \sum w_i^2 = \frac{\left[E\left({R_i}^2 {S_i}^2\right) - E(R_i S_i)^2\right]}{M}$$

and since ${R_i}^2 = 1$,

$$Var(P) = \frac{D^2(1 - IC^2)}{M}.$$

With the variance and return thus described, the ratio of excess return to excess return *volatility*, is equal to:

$$IR = IC \times \sqrt{M} \times TAS \times \frac{1}{\sqrt{1 - IC^2}}.$$

If the IC is not large (say 0.1 at most) then dividing by $\sqrt{1 - IC^2}$ will have only a small order effect (at worst altering an information ratio below 1 in the third decimal place), which completes the derivation.

APPENDIX B: INDEX VERSUS EQUAL-WEIGHTED AVERAGES

As defined earlier, the capacity-adjusted dispersion for a weighted collection of market segments is equal to their dispersion times their average size. The calculation of the two components, dispersion and size, is as follows.

Dispersion is index weighted in the sense that for a single period and without any annualization,

$$Dispersion = \sqrt{\sum w_i (C_i - P)^2}$$

Where w_i is the benchmark weight of the i^{th} constituent at the start of the period, C_i is the return of the i^{th} constituent over the period, and P is the benchmark return, equal to the weighted sum of constituent returns.

The average size of constituents is also a benchmark-weighted average,

$$Average\ Size = \sum w_i F_i$$

Where F_i is the free-float market capitalization of the i^{th} constituent (taken at the start of the period). If the benchmark is capitalization weighted, and the total benchmark capitalization is represented by TM, then each F_i is equal to $w_i TM$ and we may write

$$Average\ Size = \sum w_i^2\, TM$$

If $M = (1/\sum w_i^2)$ is the effective number of constituents in the market benchmark,[19] then it is immediately clear that we can rewrite the average size as the total benchmark capitalization divided by the effective number of constituents.

The notion of the effective number of constituents has intuitive applications. Consider the simple example of a market with five stocks as shown in Exhibit B1.

EXHIBIT B1: ILLUSTRATION OF THE EFFECTIVE NUMBER OF CONSTITUENTS IN FIVE-STOCK PORTFOLIOS

STOCK	CAP WEIGHT (%)	EQUAL WEIGHT (%)	CAP WEIGHT SQUARED (%)	EQUAL WEIGHT SQUARED (%)
A	90.0	20.0	81.0	4.0
B	2.5	20.0	0.1	4.0
C	2.5	20.0	0.1	4.0
D	2.5	20.0	0.1	4.0
E	2.5	20.0	0.1	4.0
		SUM OF SQUARES (%)	81.3	20.0
		1/SUM OF SQUARES	1.23	5.00

Source: S&P Dow Jones Indices LLC. Table is provided for illustrative purposes.

Stock A is gargantuan compared to stocks B-E and so dominates a cap-weighted index of the market. If we square the capitalization weights, add them up, and take the inverse, we can compute the effective number of positions of 1.23. The intuition behind the 1.23 is that although there are five names in the universe, four of them don't matter much; effectively we have 1.23 stocks. (On the other hand, if we assume that the index is equally weighted, then the effective number of stocks is the same as the actual number.)

As noted above, the capacity in our computation of capacity-adjusted dispersion is the index's total capitalization divided by the effective number of constituents. However, the computation of dispersion also relies on capitalization weighting. It's legitimate to wonder if we have capitalization-weighted once too often (that is, twice instead of once).

The capitalization weighting in the dispersion calculation reflects the fact that most indices are themselves capitalization weighted. If a manager takes active positions in proportion to capitalization, his excess rewards or losses versus the benchmark (in percentage terms) should have a capitalization weighting applied to their distribution.

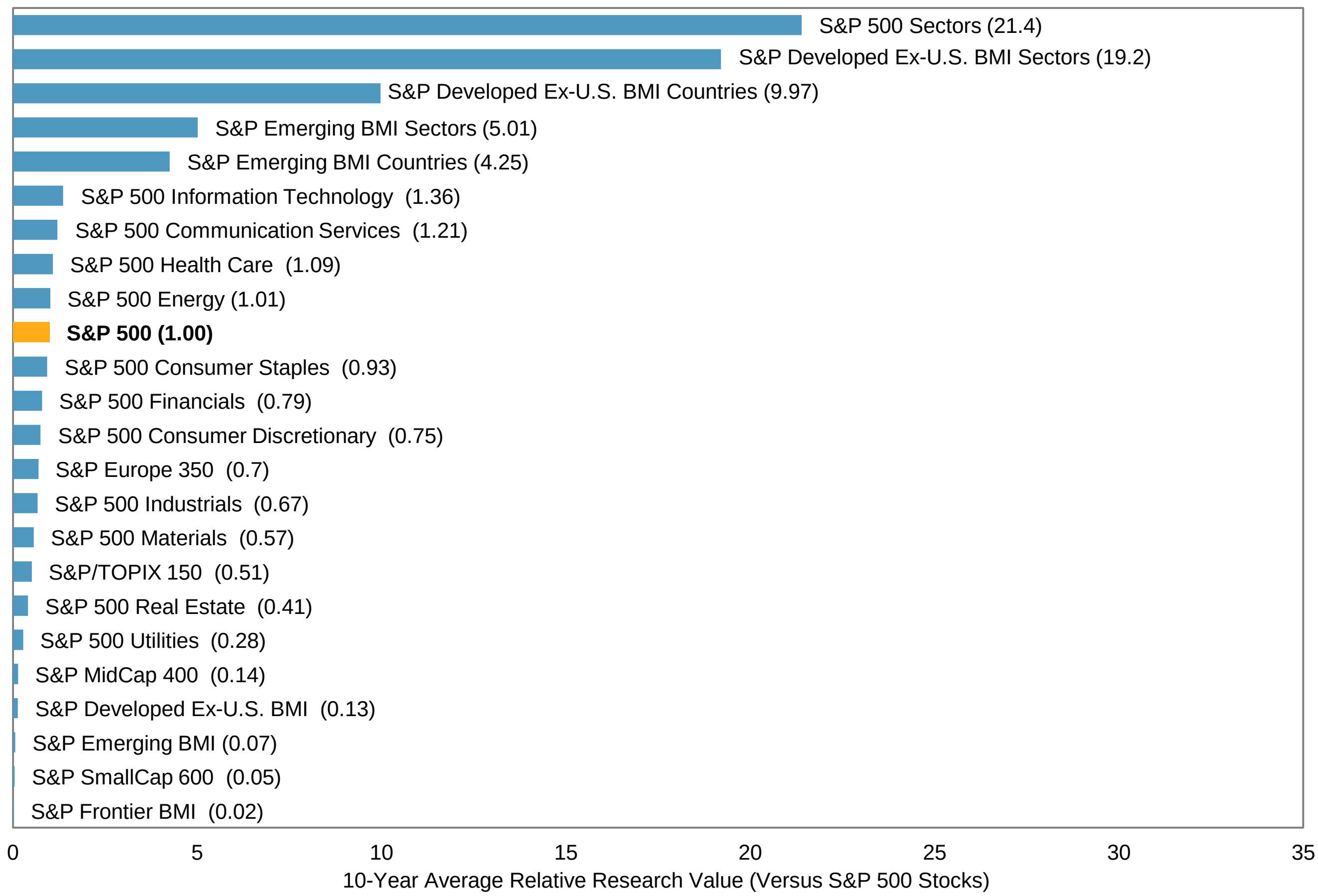
Moreover, in most practical circumstances, and particularly for broader-based indices, if managers are faced with a per-security cost of research, they are likely to pay for and implement a more limited set of overweights and underweights and—given the higher capacity they can take in response—are likely to be biased toward recommendations in constituents with higher benchmark weights.

[19] The effective number is the reciprocal of the Herfindahl-Hirschman Index concentration measure, and both statistics have become part of the literature of diversification in financial theory. The two definitions appear to have been developed independently; the latter in the familiar context of portfolio concentration via Albert Hirschman's "The Paternity of an Index" (*The American Economic Review*, Vol. 54, No. 5, September 1964, p. 176), the former as a measure of the effective number of political parties with varying numbers of parliamentary seats (Laakso, M. and R. Taagepera, "The 'Effective' Number of Parties: A Measure with Application to West Europe," *Comparative Political Studies*, 12:1, April 1979).

Our analysis imposes several stylistic assumptions on the potential manner in which managers might overweight or underweight securities in general. Implicit in the manner in which we calculated the capacity-adjusted dispersion, we assume that managers are more likely to 1) make active bets in larger constituents than smaller constituents, and 2) take larger absolute active positions in larger constituents. Further research might be conducted to see if either of these two assumptions are confirmed in the empirical data of actual active positions held by active portfolios, but as indicated in this appendix, different types of active behavior might be modeled simply through making variations in the weights used to compute average size and cross-sectional dispersion.

In order to demonstrate the potential impact of the choice between the effective and actual number of constituents, Exhibit B2 offers an alternative version of Exhibit 6, supposing capacity were computed via an equal-weight average of constituent sizes.

Exhibit B2: Capacity-Adjusted Dispersion Under Equal-Weight Average Size Calculations



Source: S&P Dow Jones Indices LLC. Data based on annualized monthly dispersion and capitalization figures from October 2008 to September 2018. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance. Past performance is no guarantee of future results.

The significant difference between the value for sectors and countries in the S&P Developed Ex-U.S. BMI displayed in Exhibit B2 contrasts to their relative similarity in Exhibit 6 and illustrates the impact of adjusting for concentration or not. As of September 2018, there were 23 countries represented in the index, eight of which with an index weight of less than 1%—and an effective number of countries equal to nine.

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The back-test period does not necessarily correspond to the entire available history of the Index. Please refer to the methodology paper for the Index, available at www.spdji.com for more details about the index, including the manner in which it is rebalanced, the timing of such rebalancing, criteria for additions and deletions, as well as all index calculations.

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