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# Smart Beta Efficiency Versus Investability: *Introducing the Cost-Adjusted Factor Efficiency Ratio*

## EXECUTIVE SUMMARY

- With an increasing number of smart beta strategies focused on the same risk factor, examining factor efficiency is one way to judge how much intended factor exposure market participants are obtaining.
- The portfolio construction approach drives factor efficiency and investability, and there is an inevitable trade-off between the two.
- To account for this trade-off, we propose a new smart beta metric for market participants—the cost-adjusted factor efficiency ratio (Ca-FER).

With an abundance of smart beta strategies, it is more important than ever to understand their underlying drivers of risk and return.

An all-inclusive descriptor of systematic, non-market-cap investing, smart beta has witnessed tremendous growth in demand, and assets under management in related exchange-traded products have soared to USD 429 billion globally.<sup>1</sup> Product launches have correspondingly been ramped up; they now number in the hundreds. Yet, despite this diversity in the types of strategies available, the vast majority aim to reap excess risk-adjusted return by targeting a limited set of risk factors—with value, momentum, volatility, dividend yield, size, and quality among the best recognized and empirically supported. It is therefore important to appreciate the underlying drivers of risk and return for these diverse strategies and draw distinctions between them, especially for those that share homogeneous objectives, as in the case of low volatility and minimum-variance strategies.

Many of the dissimilarities are rooted in differences in portfolio construction, the effects of which can be challenging to decipher, even when the mathematical properties inherent in them are well understood. The difficulty may be further compounded by market participants' inclination to judge strategies based on the strength of ex-post risk-adjusted performance; while that tendency can help inform the decision-making process, it has potential limitations. For one, there is a dearth of live performance, and a large portion of the advertised ex-post returns for these strategies include back-tested simulations, which may not be repeated once the strategies are launched. Factor performance can also vary immensely

<sup>1</sup> As of June 2016, [www.etfgi.com](http://www.etfgi.com).

over short periods of time, and achieving an inferior risk-adjusted performance does not automatically signify that the strategy is ineffectual in harvesting the factor in question. By the same token, attaining a superior ex-post return does not necessarily translate into the strategy's effectiveness, because the return may come from factors other than the ones being aimed for. For this reason, it may be better to evaluate smart beta strategies against their capacity to provide high, persistent exposures to the selected factor and view them from both a risk and return perspective.

Increasing factor exposure is generally desirable, *ceteris paribus*, but it often comes at a cost, both financial and non-financial.

Increasing factor exposure is generally desirable, *ceteris paribus*, but it often comes at a cost, both financial and non-financial. Financial costs may include expenses arising from more regular rebalancing or purchasing less liquid stocks. Non-financial costs may involve meeting certain legal requirements, such as being compelled to disclose the shareholding in companies that are held in an investment portfolio, or being obligated to take over the entire company in the most extreme case. Regulatory requirements aside, enhancing factor exposure may pose considerable operational challenges, particularly in cases where factor exposure is enhanced through the use of leverage or short selling; these techniques are also incredibly complex. Therefore, there is a trade-off between increasing exposure and greater investability. In this paper, we focus on the financial tradeoff between achieving greater targeted exposure and ensuring investability. To this end, we introduce a new smart beta metric—the cost-adjusted factor efficiency ratio (Ca-FER)—as an adaptation to the one Hunstad and Dekhayser (2015) proposed in order to acknowledge such a tradeoff. We demonstrate the potential application of this metric through detailed case studies analyzing single-factor and multi-factor long-only portfolios, which are built using both ranking- and optimization-based construction techniques.<sup>2</sup> We further examine the merits of building concentrated factor portfolios vis-à-vis diversified portfolios.

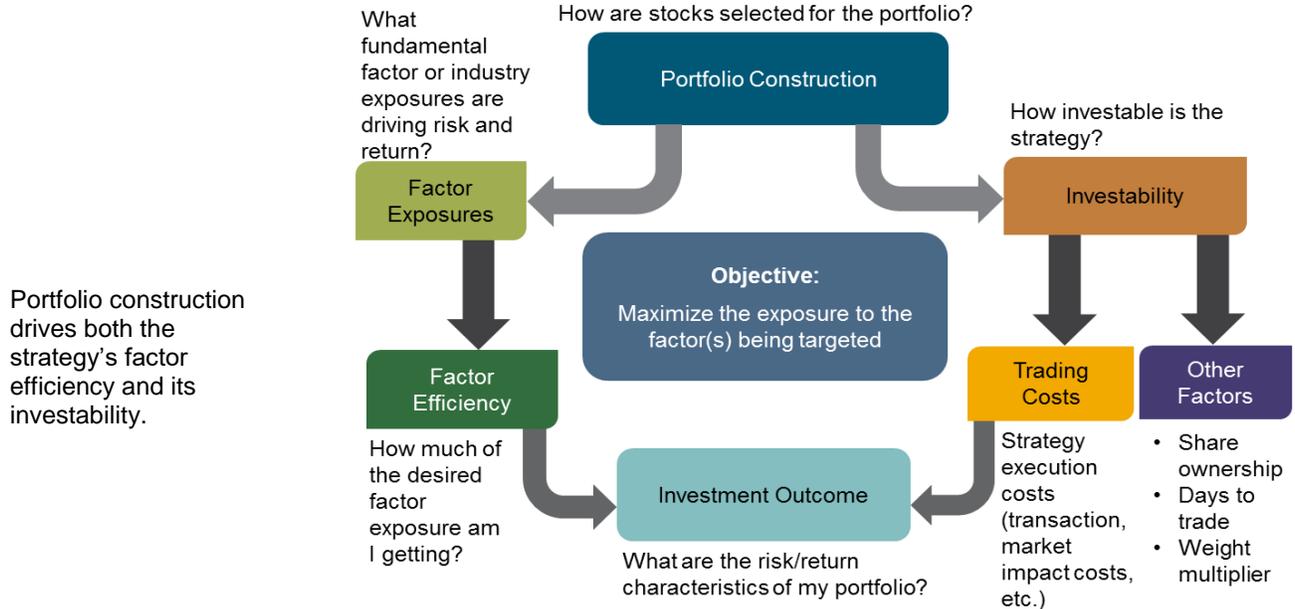
## THE CONNECTION BETWEEN EFFICIENCY, INVESTABILITY, AND PORTFOLIO CONSTRUCTION

To achieve exposure to a particular factor, a portfolio must deviate from a market-cap weighting. This may result in unintended exposures to secondary factors that come with moving away from the benchmark (Ung and Luk, 2016). A case in point is dividend-yielding strategies, which frequently see a noticeable increase in weight in both the finance and utilities sectors. If these weights were unconstrained, the portfolios would be dominated by companies in these sectors. Therefore, the resulting risk-adjusted performance would be predominantly derived from large sector bets, rather than the desired factor. Conversely, placing multiple rigid

<sup>2</sup> Ranking-based construction techniques involve converting fundamental variables into standardized z-scores, which are then used to directly weight or tilt the benchmark weights of eligible securities. In these approaches, optimization via a risk model is not used.

constraints in order to avoid divergence from the benchmark may impede the strategy’s ability to reap the return of the factor. This may engender a drop in the transfer coefficient,<sup>3</sup> as potentially useful information may not be fully leveraged. It follows from this that portfolio construction drives the amount of desired factor exposure—efficiency—and how implementable and investable the strategy is—investability (see Exhibit 1). Thus, there is a balancing act between efficiency and investability that needs to be carefully managed (see Exhibit 2).

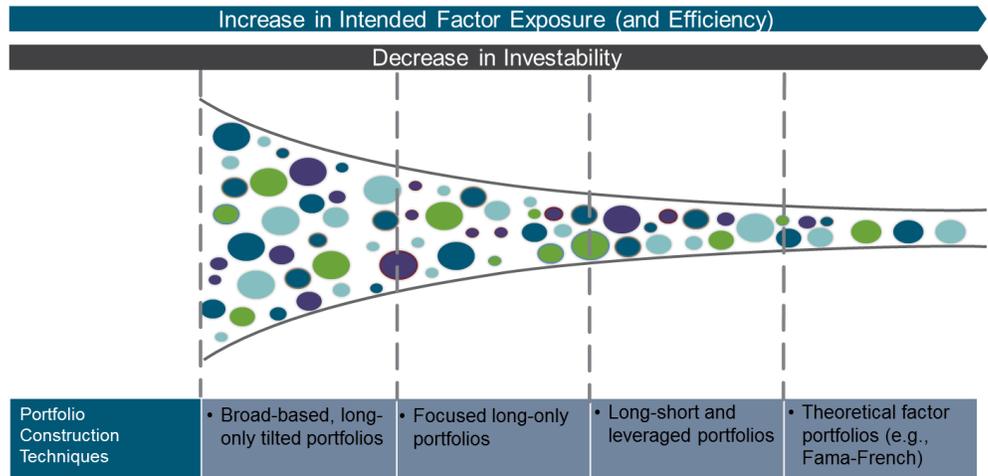
**Exhibit 1: Portfolio Construction Drives Efficiency and Investability**



Source: S&P Dow Jones Indices LLC. Chart is provided for illustrative purposes.

<sup>3</sup> The transfer coefficient is an ex-ante metric commonly used by quantitative managers to measure the correlation between benchmark-relative (active) weights and forecast residual returns. It serves to measure how much of the trading signal is transferred through to active weights.

**Exhibit 2: Tradeoff Between Factor Efficiency and Investability**



Source: S&P Dow Jones Indices LLC. Chart is provided for illustrative purposes.

Authors such as Hunstad and Dekhayser (2015) advocate the use of the factor efficiency ratio (FER) to quantify efficiency and facilitate comparison of different beta strategies, given the array of similar strategies in existence (see Equation 1). The ratio is calculated as follows, where  $FER$  is the factor efficiency ratio,  $FC_d$  is the active risk contribution of  $d$  desired factors, and  $AR_p$  is the total active risk of the portfolio with respect to the benchmark.

$$FER = \frac{\sum FC_d}{AR_p - \sum FC_d} \quad (1)$$

The ratio analyzes how much active risk comes from primary (intended) exposure, as opposed to secondary (unintended) exposure, and it can therefore be regarded as a factor “purity”<sup>4</sup> measure. Of course, factor efficiency may not be something that has to be sought, and a thorough dissection of a strategy’s exposures may suffice for those to whom factor efficiency is not a pertinent consideration. On the other hand, for those that wish to obtain the maximum possible amount of exposure to the selected factor, efficiency may indeed form part of a broader assessment that determines which strategy is most felicitous.

Another consideration that is distinct but related to factor efficiency has to do with the investability associated with increasing efficiency. One of the many facets of investability is the trading cost incurred with the implementation of smart beta strategies, which gives rise to an efficiency-investability tradeoff. Accordingly, we have introduced Ca-FER (see Equation 2), which recognizes this. It is calculated as follows.

$$Ca - FER = \left(1 - \frac{\sum C_i}{TE}\right) \times FER \quad (2)$$

Either the FER must increase or the total trading cost per unit tracking error must decrease (or both) in order for the Ca-FER to go up.

<sup>4</sup> Factor “purity” here does not mean absolute (theoretical) purity. It refers to the amount of active risk that comes from intended (primary) exposures with respect to the amount of active risk from unintended (secondary) exposures; i.e., purity in the context of “real” index portfolios.

$Ca - FER$  is the cost-adjusted factor efficiency ratio,  $\sum C_i$  represents the total execution costs (e.g., transaction cost, market impact cost, etc.) and is defined as the sum of  $i$  cost components associated with the implementation of the strategy, expressed as a percentage of assets under management.  $TE$  is the tracking error of the strategy with respect to its benchmark, and  $FER$  is the factor efficiency ratio.

From this, it follows that either the FER must increase or the total trading cost per unit tracking error must decrease (or both) in order for the Ca-FER to go up. The mathematical expression that precedes FER in the Ca-FER calculation can therefore be interpreted as a “discount” applied to the factor efficiency ratio. Stated differently, any growth in factor efficiency is only worthwhile if it exceeds the total execution costs connected with it, on the basis that costs are the main issue at stake.<sup>5</sup> The accuracy of Ca-FER depends upon the reliable measurement of total execution costs, and market participants should therefore feel free to choose their preferred cost estimation method.

For analytical tractability and simplicity, we have opted in the following case studies to use the mathematical framework suggested by Aked and Moroz<sup>6</sup> to estimate the market impact cost<sup>7</sup> from rebalance trading. The aim here is not to pass judgement on the merit of their approach; rather, because their framework is mathematical in nature, it allows us to better understand the underlying features of the rival strategies. The crux of the matter is that this analysis is conducted and that the selected method is applied consistently across competing strategies.

It is also worth emphasizing that the following analyses serve to illustrate how Ca-FER can be used to pick the most exposure-efficient strategy from an array of choices. It is not meant to prescribe the “best” portfolio construction approach, as this would be contingent upon the market participant’s risk appetite, constraints, and individual preferences.

## CASE STUDY 1: DIVERSIFIED SINGLE-FACTOR PORTFOLIOS

Regardless of how they are created, smart beta indices aim to harvest specific factor exposures, but many of them have ancillary sector and factor exposures as well. Their implementation costs are equally likely to be tangibly different. To demonstrate this, we examined a number of strategies that were built using ranking-based and optimization-based

<sup>5</sup> In addition to trading costs, there may be other necessary considerations. In the UK, the City Code on Takeover and Mergers states that any public company that owns, as a shareholder or through a holding of financial instruments, 3% or more of the company must, within two trading days, disclose the amount of holdings to the company concerned. <http://www.thetakeoverpanel.org.uk/>.

<sup>6</sup> This is done through studying the strategy’s weighted average market cap, portfolio turnover, the level of tilt toward less liquid stocks, and the amount of assets under management, among other criteria. For further details, please refer to Appendix I and the original paper, “The Market Impact of Passive Trading” (Aked and Moroz, 2015).

<sup>7</sup> Total execution costs should be used, where possible.

Choosing the “best” smart beta approach involves balancing investment objectives and specific constraints.

techniques, were subjected to a variety of constraints, and had the ultimate goal of creating diversified portfolios that target the value factor within the U.S. large-cap stock universe.

All the strategies drew on the entire universe's standardized z-scores, which were derived from the average of three fundamental ratios: book-to-price, revenue-to-price, and earnings-to-price. Portfolios that were assembled using ranking-based construction methodologies (Portfolios 1 to 3) attributed weights to companies that were commensurate to their overall value z-scores, adjusted by market cap. As for portfolios built by optimization approaches (Portfolios 4 to 7), the same set of value z-scores was used, but the weights of companies were optimized in such a way that the strategies remained diversified<sup>8</sup> and contained the most significant level of value characteristics, so long as the constraints and risk parameters were observed. In all cases, the portfolios were reconstituted every six months between December 2009 and April 2016. Exhibit 3 summarizes the major characteristics of the seven value portfolio construction approaches.

Upon examination of the annualized return per unit risk, all seven portfolios seemed very much alike, with a narrow spread between the best- and worst-performing portfolios. This was particularly true for Portfolios 1 and 2, which, by virtue of the similar manner in which they were assembled, possessed many common attributes. Both portfolios made use of the entire universe of stocks in the benchmark, but Portfolio 2's sector weights were mandated to be kept the same as those in the benchmark at each portfolio rebalance. This sector neutrality requirement not only increased the amount of momentum risk<sup>9</sup> and reduced the amount of industry risk, but it also seems to have raised the amount of stock-specific risk in the strategy. Nevertheless, it does not appear to detract from the amount of active value risk. Additionally, the market impact costs experienced by trading the two portfolios were comparable, even though Portfolio 1 had an average overweight of 6.4% in financials and an average underweight of 4.3% in information technology (see Exhibit A2.1 in Appendix 2). In contrast, Portfolio 3 experienced a market impact cost that was about six times larger than Portfolio 1. This was possibly because the former portfolio contained more small-cap stocks than the latter portfolio, resulting in an overall lower-weighted average market cap in Portfolio 3 (two-thirds lower than that in Portfolio 1). This was likely the product of assigning higher weights to companies based on z-scores alone, which severed the relationship between the weighting scheme and market cap. Other unintended consequences were that Portfolio 3 took the longest time to trade and generated the lowest FER (0.058), because the portfolio's active risk was dominated by market beta and size, rather than value risks, which accounted for less than 5% of its total tracking error.

The sector neutrality requirement reduced the amount of industry risk, but it also seems to have raised the amount of momentum and stock-specific risk in the ranking-based value portfolio.

<sup>8</sup> The target tracking error for all the optimized portfolios is set at 1.5% per year.

<sup>9</sup> See Exhibit A2.1

Exhibit 3: Comparison of Different Ranking- and Optimization-Based Diversified Value Portfolios Within the S&amp;P 500 Universe

CONSTRUCTION AND ANALYTICS		RANKING-BASED PORTFOLIO CONSTRUCTION			OPTIMIZATION-BASED PORTFOLIO CONSTRUCTION			
		PORTFOLIO 1	PORTFOLIO 2	PORTFOLIO 3	PORTFOLIO 4	PORTFOLIO 5	PORTFOLIO 6	PORTFOLIO 7
Portfolio Construction	All Constituents in Strategy?	Yes	Yes	Yes	No	No	No	No
	Weighting	Market-cap-adjusted value <sup>d</sup> z-scores	Market-cap-adjusted value <sup>d</sup> z-scores	Value <sup>d</sup> z-scores	Optimized based on value <sup>d</sup> z-scores	Optimized based on value <sup>d</sup> z-scores	Optimized based on value <sup>d</sup> z-scores	Optimized based on value <sup>d</sup> z-scores
	Maximum Weight	4 times benchmark weight	4 times benchmark weight	4 times benchmark weight	4 times benchmark weight	4 times benchmark weight	4 times benchmark weight	15 times benchmark weight
	Other Constraints (With Regard to S&P 500)	-	Sector neutrality <sup>g</sup>	-	Sector neutrality <sup>g</sup>	Unintended factors neutrality <sup>a</sup>	Sector <sup>g</sup> and unintended factors neutrality <sup>a</sup>	Sector <sup>g</sup> and unintended factors neutrality <sup>a</sup>
Main Characteristics	Annualized Return per Unit Risk	0.892	0.917	0.960	0.958	0.910	0.968	0.990
	Largest Active Stock Weight (% With Regard to S&P 500) <sup>b</sup>	1.197	0.874	-3.245	-0.756	1.050	-0.787	1.182
	Weighted Number of Days to Trade <sup>c</sup>	0.244	0.237	0.334	0.320	0.347	0.337	0.397
	Weighted Stock Ownership (%) <sup>c</sup>	0.031	0.030	0.058	0.043	0.045	0.045	0.054
Risk Analysis	FER	0.262	0.268	0.051	0.488	0.616	0.582	0.693
	Tracking Error (% per Year With Regard to S&P 500)	1.899	1.360	3.645	1.435	1.517	1.412	1.431
	Value Risk (% Tracking Error) <sup>d</sup>	20.7	21.1	4.9	32.8	38.1	36.8	40.9
	Common Factor Risk, Excluding Value and Industry (% Tracking Error) <sup>e</sup>	39.0	35.0	64.2	34.3	18.4	20.3	18.5
	Industry Risk (% Tracking Error) <sup>e</sup>	13.5	8.1	9.2	9.6	18.7	12.6	11.1
	Stock-Specific Risk (% Tracking Error)	26.7	35.7	21.7	23.3	24.8	30.4	29.4
Cost <sup>f</sup>	Market Impact Cost (%) <sup>f</sup>	0.055	0.059	0.318	0.394	0.838	0.306	0.454
	Weighted Average Market Cap (USD Millions) <sup>f</sup>	668.911	703.769	238.038	717.694	731.600	731.910	738.612
	Portfolio Turnover (% per Year) <sup>f</sup>	1.000	1.136	1.785	5.011	9.936	3.595	4.327
	Illiquidity Tilt <sup>f</sup>	0.011	0.011	0.012	0.016	0.019	0.018	0.022
Ca-FER		0.254	0.256	0.047	0.354	0.276	0.456	0.474

Portfolios 1-7 are hypothetical portfolios.

Source: S&P Dow Jones Indices LLC and Northfield Information Services, Inc. Data from Dec. 31, 2009, to April 31, 2016. Past performance is no guarantee of future results. Table is provided for illustrative purposes. Results for optimized portfolios will vary for different risk parameters and optimization engines. Cells shaded light teal represent the highest number in that row, and cells shaded light grey represent the lowest number. NOTE: <sup>a</sup> factors neutrality means that all fundamental factors have to be within 0.05 standard deviations relative to the S&P 500; <sup>b</sup> largest active stock weight means the largest absolute tilt away from the benchmark; <sup>c</sup> calculated based on a floating portfolio size of 0.025 of total S&P 500 market value (or approximately USD 3.58 billion over the period); <sup>d</sup> value risk is the sum of active risk from price-to-earnings, price-to-book, and price-to-sales; <sup>e</sup> the total active risk derived from factors other than value risks, industry risks, and stock-specific risks, as defined by Northfield; <sup>f</sup> is defined as the cost based on model described in Aked and Moroz (2015), please refer to Appendix 2; <sup>g</sup> sector neutrality means that portfolio sector weights must be the same as the S&P 500 at rebalance.

The additional market impact cost incurred by neutralizing unintended factor exposures more than wiped out the benefit gained from more generous value exposures, leading to an average-ranking Ca-FER.

Conversely, Portfolios 4 to 7 had the highest level of active value risk. This did not come as a surprise, because these portfolios were optimized, and the optimization process was designed to achieve maximum value exposure, albeit with various constraints. For instance, Portfolio 4 had significant value exposure, but it also had a large momentum exposure. This may be ascribed to its sector-neutral conditionality, which effectively aligned the sector weight of the strategy to that of the benchmark every six months. This also underscored the difference in the construction of Portfolios 4 and 5; although the latter also attempted to augment value exposures, it imposed the condition of neutrality on other unintended exposures, but not on sectors. As a result, Portfolio 5 had the greatest amount of industry risk among all the portfolios analyzed. Furthermore, it had the highest market impact cost per unit of tracking error, on account of more frequent trading,<sup>10</sup> which was brought about by the necessity of realigning unstable secondary exposures to those of the benchmark. This additional cost more than wiped out the benefit gained from more generous value exposures, leading to a Ca-FER that merely ranked average among all the portfolios studied.

Finally, Portfolios 6 and 7 intended to strike a balance, in that the restrictions of sector and unintended-exposure neutrality must be respected. This resulted in lower market impact cost per unit tracking error, although both of these portfolios still had high value exposures. Indeed, there was little that set the two apart, even though the weight constraint for Portfolio 6 was much more rigid than for Portfolio 7, which allowed a stock to have a maximum weight of no greater than fifteen times its benchmark weight.<sup>11</sup>

Nevertheless, caution should be exercised in generalizing the results of this case study, as it would be an oversimplification to conclude that the “best” way to build a value portfolio is by adopting the technique used in Portfolio 7. The challenge for the construction of any investment portfolio is to operate within the confines and constraints prescribed by its ultimate beneficiaries, in terms of risk, return, and other specific preferences.

## **CASE STUDY 2: CONCENTRATED SINGLE-FACTOR PORTFOLIOS**

Next, we probed the merits of constructing high-conviction factor portfolios. Akin to other investment portfolios, factor portfolios can be either diversified or concentrated. As previously mentioned, they can be built using ranking- or optimization-based techniques. One school of thought argues that factor

<sup>10</sup> The market impact cost model developed by Aked and Moroz (2015) assumes that turnover resulting from reweighting securities is costlier than that resulting from adding or dropping securities from the strategy. For this reason, turnover for Portfolio 5 is substantially higher than for Portfolio 1, which has no adds or drops because all securities must be included in the strategy at all times.

<sup>11</sup> See Exhibit A2.1 in the appendix for details.

Selecting the top momentum stocks generated more active risk from momentum, but it was done at the expense of marginally lower efficiency.

portfolios should be driven by high conviction and target only those small number of stocks that convincingly exhibit coveted factor attributes. On the surface, this stance appears to hold water, since deviating from benchmark weights, or having some tracking error, is a requisite to harvesting risk factors. The points of contention then become how concentrated portfolios ought to be and whether factor exposures are best delivered via high-tracking-error, concentrated portfolios. To test this, we compared two ranking-based momentum portfolios that used standardized momentum z-scores, calculated as the risk-adjusted, 12-month price change, excluding the month of rebalance. Construction of Portfolio 8 involved preserving all the constituents in the benchmark and merely tilting benchmark weights by their corresponding z-scores; Portfolio 9 was identical in all respects, except that only securities in the top one-half of z-scores were retained. Both portfolios were rebalanced every six months.

Exhibit 4 highlights the principal features of the two portfolio construction approaches. Because Portfolio 9 selected only the top 50% of stocks in terms of momentum, it had a higher tracking error than Portfolio 8 (2.7% per year, versus 4.5%). This contributed to larger biases away from the benchmark, a greater maximum stock weight multiplier, and a longer average time to trade the entire portfolio.

From a factor exposure standpoint, even though Portfolio 9 had a greater tracking error and concentrated solely on the highest-momentum stocks, the relative amount of risk resulting from the momentum factor, as a percentage of tracking error, is approximately comparable to Portfolio 8's. Even so, Portfolio 9 produced a larger amount of risk from momentum on an absolute basis. Put differently, Portfolio 9 generated more active risk from momentum, but it was done at the expense of marginally lower efficiency, when viewed from a total-risk perspective. Also of note, industry (sector) risk accounted for a small amount of tracking error in both strategies, which may seem to defy logic, as there is an oft-held belief that momentum strategies would be driven predominantly by sector risk.

While the intended factor efficiency for both strategies is comparable, the way in which momentum risks are generated is not. Indeed, the market impact cost per unit tracking error of Portfolio 9 was nearly three times higher than that of Portfolio 8. Additionally, although the former portfolio has a higher average market cap, its significant portfolio churn rate and strong bias toward less liquid stocks increase the overall market impact cost per unit tracking error, leading to a markedly inferior Ca-FER.

Exhibit 4: Comparison of Momentum Portfolios Within the S&amp;P 500 Universe

CONSTRUCTION AND ANALYTICS		DIVERSIFIED TILTED PORTFOLIO CONSTRUCTION	CONCENTRATED PORTFOLIO CONSTRUCTION
		PORTFOLIO 8	PORTFOLIO 9
Portfolio Construction	Average Number of Constituents	500	250 (Top 50% by momentum scores)
	Weight Scheme	Weighting	Market-cap adjusted momentum z-scores
		Maximum Weight	4 times benchmark weight
Main Characteristics	Annualized Return per Unit Risk	0.930	0.909
	Largest Active Stock Weight (With Regard to S&P 500) <sup>a</sup>	3.046	4.819
	Weighted Number of Days to Trade <sup>b</sup>	0.280	0.418
	Weighted Stock Ownership (%) <sup>b</sup>	0.000	0.001
	Maximum Weight Multiplier (With Regard to S&P 500)	3.046	4.819
	Largest Sector Overweight (%)	Information Technology 2.987	Information Technology 5.324
	Largest Sector Underweight (%)	Energy 3.183	Energy 4.900
Risk Analysis	FER	0.583	0.567
	Tracking Error (% per Year With Regard to S&P 500)	2.741	4.463
	Momentum Risk (% Tracking Error)	36.8	36.2
	Total Common Factor Risk, Excluding Momentum and Industry (% Tracking Error)	24.7	25.7
	Industry Risk (% Tracking Error)	11.0	11.5
	Stock-Specific Risk (% Tracking Error)	27.4	26.6
Cost <sup>c</sup>	Market Impact Cost (%) <sup>c</sup>	0.996	4.424
	Weighted Average Market Cap (USD Millions) <sup>c</sup>	869.992	952.484
	Portfolio Turnover (% per Year) <sup>c</sup>	16.800	49.500
	Illiquidity Tilt <sup>c</sup>	0.015	0.025
Ca-FER		0.371	0.005

Divergence from the benchmark may be indispensable, but portfolio concentration alone may not yield more exposure to the desired factor.

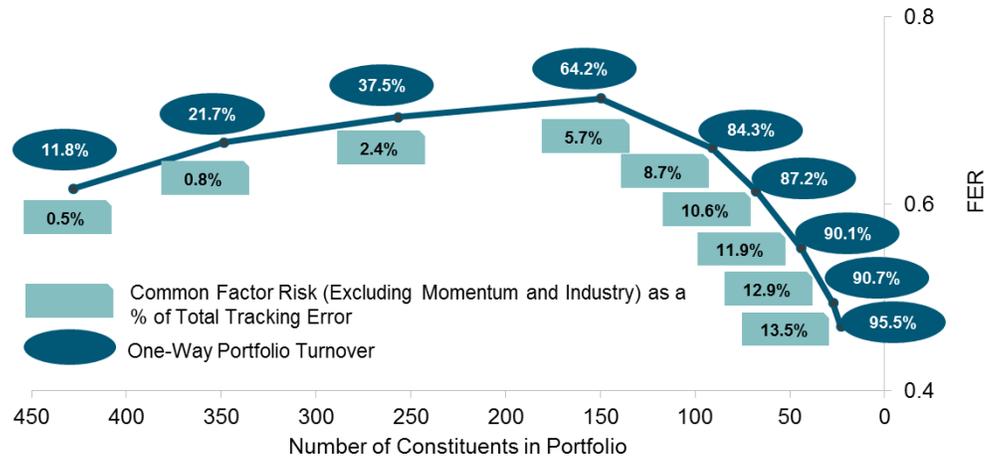
Portfolios 8 and 9 are hypothetical portfolios.

Source: S&P Dow Jones Indices LLC and Northfield Information Services, Inc. Data from Dec. 31, 2009, to April 30, 2016. Past performance is no guarantee of future results. Table is provided for illustrative purposes. Results for optimized portfolios will vary for different risk parameters and optimization engines. Cells shaded light teal represent the highest number in that row. NOTE: <sup>a</sup> Largest active stock weight means the largest absolute tilt away from the benchmark; <sup>b</sup> Calculated based on a floating portfolio size of 0.025% of total S&P 500 market value (or approximately USD 3.58 billion over the period); <sup>c</sup> Defined as the cost based on model described in Aked and Moroz (2015), please refer to Appendix 2.

Divergence from the benchmark may be indispensable, but portfolio concentration alone may not yield more exposure to the desired factor, in terms of the percentage of active risk taken on a total basis. Determining precisely how many stocks are needed to produce the highest level of factor efficiency is challenging a priori and is based on the portfolio construction process. However, there is a compromise between the level of portfolio concentration and the efficiency of a given factor exposure. Exhibit 5 indicates how the level of FER in relation to the momentum factor, portfolio turnover, and amount of risk derived from non-momentum common factors changed for portfolios with a varying number of stocks. All these

sample portfolios had the same common goal: to boost the amount of momentum exposure as far as possible by conducting optimizations via the Northfield U.S. Fundamental Equity Risk Model. In all cases, the risk parameters used in the optimizations were identical, aside from the number of stocks in each portfolio.

**Exhibit 5: Level of FER and Portfolio Turnover in Concentrated Momentum Portfolios Within the S&P 500 Universe**



Intended factor exposure decreased and unintended factor exposure increased as portfolios became more concentrated.

Source: S&P Dow Jones Indices LLC and Northfield Information Services, Inc. Data from Dec. 31, 2009, to April 30, 2016. Past performance is no guarantee of future results. Chart is provided for illustrative purposes. Results for optimized portfolios will vary for different risk parameters and optimization engines.

The level of momentum exposure initially increases with fewer stocks in the portfolio, and this comes with a higher portfolio turnover rate, as the stocks that do not contribute to higher momentum exposure or that are risk-inefficient in the wider portfolio context are removed and supplanted by alternative ones. In this stylized example, the momentum FER seems to peak at around 150 stocks, at which point it rapidly collapses; meanwhile, the portfolio turnover rate continues to climb<sup>12</sup> as the portfolio becomes more concentrated. Active risk derived from exposure to other common factors also rises gradually with portfolio concentration and eventually overtakes the amount of risk derived from exposure to momentum. This implies that a strong conviction approach—or a substantial degree of portfolio concentration—decreases desired factor efficiency, as the portfolio racks up other unintended fundamental factor risks, to the detriment of the targeted factor risk. Consequently, high-conviction portfolios tend to experience two adverse effects—falling efficiency in the targeted factor and prohibitive costs caused by excessive portfolio turnover.

<sup>12</sup> It is possible to take measures to lower tracking error, but this often comes at the expense of reduced intended factor exposure.

### CASE STUDY 3: MULTI-FACTOR PORTFOLIOS

Lastly, we turn to discuss the diversity of ways to construct multi-factor portfolios. It is worth noting that multi-factor strategies are a means and not an end, and the suitability of a particular approach depends on the ultimate objective of the index. In this case study, we opted to blend two single factor portfolios, with the aim of smoothing out the volatility that can come with exposure to either factor on its own. To do this, we confined our analysis to the “index of indices” method;<sup>13</sup> that is, the underlying factor indices were created separately, and then an equal weighting was accorded to each of them, thereby assembling the multi-factor portfolios. As an illustration, we simulated two multi-factor portfolios that blended value and momentum factors. Portfolio 10 gave an equal weight to value-tilted and momentum-tilted portfolios, both of which kept all the constituents in the benchmark and applied a weighting scheme that adjusted their market-cap weights by their relative factor scores. On the other hand, Portfolio 11 assigned 50% of the portfolio to an optimized value strategy and 50% to an optimized momentum strategy. Each factor optimization was then conducted separately and on the proviso that sector neutrality was maintained.

Multi-factor portfolios blending the same factors can produce a markedly different mix of factor exposures, depending on the portfolio construction process.

As seen in Exhibit 6, the maximum stock weight multiplier for both blended-factor portfolios was lower than that for their underlying single-factor portfolios. Besides, both portfolios gained from a reduction in tracking error by blending the two factors, with Portfolio 11 benefitting the most. In all, common factor risks, excluding industry risk, accounted for a larger portion of the tracking error in Portfolio 11 than in Portfolio 10, and Portfolio 11 also had a more robust (momentum and value) FER. This is not surprising, because the optimizations underpinning the former portfolio were designed to give maximum exposure, within a risk-efficient context, to the individual primary factors in question. While both portfolios tilted toward financials, the extent of the tilt was quite distinct, in that Portfolio 11 only had an average overweight of 0.78% in financials and Portfolio 10 had an overweight of 2.4%. The same cannot be said about sector underweights; Portfolio 10 tilted away from the technology sector the most, while Portfolio 11 deviated furthest from the materials sector.

More remarkable is the way in which value and momentum exposures were distributed across the two strategies. Portfolio 11 had the more “balanced” exposure to momentum and value, which accounted for 23.8% and 17.9% of the total tracking error, respectively. In contrast, Portfolio 10 had a stronger bias toward momentum than value, with the former accounting for just under twice the level of tracking error as the latter. Around 55%-60%

<sup>13</sup> Depending on the ultimate investment objective, different approaches may be used to construct multi-factor portfolios, either as an “index of indices” approach or scoring at the stock level. Scoring at the stock level can also be performed in a variety of ways, which may include giving equal consideration to each of the smart beta factors or placing more emphasis on certain factors via different weighting schemes.

of the tracking error of both portfolios could be attributed to common factor risks, excluding industry risk.

Regarding market impact cost, Portfolio 10 experienced about one-third the cost of Portfolio 11 on a per unit tracking error basis, chiefly as a result of lower portfolio turnover and a higher weighted average market cap in the portfolio. Overall, although Portfolio 11 had a greater FER than Portfolio 10, its higher market impact cost per unit tracking error more than offset the benefit gained from increasing intended factor exposures. This explains why the Ca-FER for Portfolio 11 was lower than that for Portfolio 10 in this stylized example.

The findings from this analysis will differ depending on how portfolios are blended and what risk parameters and constraints are used in the ranking and optimization procedures.

**Exhibit 6: Comparison of Stylized Multi-Factor Portfolios**

	CONSTRUCTION AND ANALYTICS	PORTFOLIO 10	PORTFOLIO 11	
Portfolio Construction	<b>Average Number of Constituents</b>	500	453	
	<b>Weight Scheme and Selection</b>	Selection	No Selection (Entire universe eligible)	Constituents retained after the optimization process by either optimized index, based on an objective tracking error of 1.5% per year and sector neutrality
		Weighting	Market-cap adjusted factor z-scores	Optimization based on factor z-scores
		Maximum Weight	4 times benchmark weight	4 times benchmark weight
Main Characteristics	<b>Annualized Return per Unit Risk</b>	0.917	0.998	
	<b>Largest Active Stock Weight</b> (With Regard to S&P 500) <sup>a</sup>	1.176	0.433	
	<b>Maximum Weight Multiplier</b> (With Regard to S&P 500)	2.399	2.011	
	<b>Largest Sector Overweight</b> (%)	Financials 2.436	Financials 0.765	
	<b>Largest Sector Underweight</b> (%)	I.T. 1.207	Materials 0.780	
Risk Analysis	<b>FER</b> (both Value and Momentum)	0.327	0.548	
	<b>Tracking Error</b> (% per Year With Regard to S&P 500)	1.252	0.793	
	<b>Value Risk</b> (% Tracking Error)	9.8	17.9	
	<b>Momentum Risk</b> (% Tracking Error)	17.0	23.8	
	<b>Total Common Factor Risk</b> , Excluding Momentum and Industry (% Tracking Error)	31.0	21.8	
	<b>Industry Risk</b> (% Tracking Error)	10.8	10.4	
	<b>Stock-Specific Risk</b> (% Tracking Error)	32.6	26.0	
Cost <sup>b</sup>	<b>Market Impact Cost</b> (%) <sup>b</sup>	0.289	0.596	
	<b>Weighted Average Market Cap</b> (USD Millions) <sup>b</sup>	767.350	712.226	
	<b>Portfolio Turnover</b> (% per Year) <sup>b</sup>	3.714	9.909	
	<b>Illiquidity Tilt</b> <sup>b</sup>	0.011	0.012	
	<b>Ca-FER</b>	0.241	0.136	

Portfolios 10 and 11 are hypothetical portfolios.

Source: S&P Dow Jones Indices LLC and Northfield U.S. Fundamental Equity Model. Data from Dec. 31, 2009, to April 30, 2016. Past performance is no guarantee of future results. Table is provided for illustrative purposes. Results for optimized portfolios will vary for different risk parameters and optimization engines. Cells shaded light teal represent the highest number in that row. Note: <sup>a</sup> Largest active stock weight means the largest absolute tilt away from the benchmark; <sup>b</sup> is defined as the cost based on model described in Aked and Moroz (2015), please refer to Appendix 2.

The higher market impact cost experienced by equal-weighting the underlying optimized portfolios more than offset the benefit gained from increasing intended factor exposures.

## CONCLUSION

Given the increasing range of smart beta indices targeting the same factor, market participants may want to conduct their own due diligence to decide which strategy best suits their objectives. One aspect that may be worth examining is the amount of active risk generated from intended exposures across various strategies; one way to do that is by using Aked and Hunstad's FER. The level of FER ultimately depends on the strategy's portfolio construction process, which also determines how investable the portfolio is. Indeed, there is often a compromise between factor efficiency and investability. To account for the financial tradeoff between factor efficiency and investability, we have proposed a new smart beta metric—Ca-FER.

There is often a compromise between factor efficiency and investability, and Ca-FER may be used to quantify this tradeoff.

With the aid of stylized examples, the first part of this paper reviewed a number of diversified factor construction approaches and concluded that optimization generally provides the largest intended factor exposure, though frequently at a higher portfolio churn rate. Whether this is beneficial depends on how much extra cost is incurred from the extra factor exposure gained. The second part of the paper examined whether high-conviction, concentrated portfolios are apt for obtaining high levels of intended factor exposures. It concluded that significant levels of portfolio concentration may lead to a drop in the FER and an increase in unintended factor exposures, as well as a meaningful hike in portfolio turnover. The paper ended by examining two variations of multi-factor portfolios, concluding that differences in sector biases, FER, and Ca-FER derive from their distinct portfolio construction processes and can produce surprising results.

Finally, it is necessary to emphasize that the aim of this research paper is to introduce the Ca-FER as a new dimension, which may be used in conjunction with other criteria that are already at the disposal of market participants, to judge smart beta portfolios. If obtaining the maximum amount of targeted exposure is one of the objectives, then, all else being equal, selecting portfolios with the highest Ca-FER may be desirable. That being said, the case studies in this research paper are stylized examples and are not meant to describe the "best" approach of targeting factor exposures, which is invariably a trade-off between competing priorities—namely, achieving the best investment outcome and observing the constraints that are placed upon the portfolio construction process.

## APPENDIX 1

Risk models are particularly instructive in helping to understand the characteristics of different strategies, as they can help explain where the performance has come from by attributing risk and return to systematic and stock-specific factors. In this research paper, we used the Northfield U.S. Fundamental Equity Risk Model, which contains 11 fundamental factors, 55 industry factors and the market beta.<sup>14</sup> The model is based on a relaxed capital asset pricing model construct and is founded on the premise that securities are correlated with the general market beta. In addition, it acknowledges that other factors are also apropos to explaining equity return. The model can be summarized as follows.

$$R_j - R_f = \beta_j [R_m - R_f] + \sum_{k=1}^{66} E_{j,k} r_k + \varepsilon_j$$

Common Factors

Long-Only Factor (Beta)      Isolated Long-Short Fundamental (11 Factors) and Industry Factors (55 Factors)      Stock-Specific Factor

where  $R_j$  is the return on asset  $j$ ;  $R_m$  is the market return;  $R_f$  is the risk-free rate (e.g., short-term U.S. Treasury Bonds);  $\beta_j$  is the market beta;  $E_{j,k}$  is the exposure<sup>15</sup> of asset  $j$  to factor  $k$  at time  $t$ ;  $r_k$  is the return (long-short) factor  $k$  at time  $t$ ; and  $\varepsilon_j$  is the idiosyncratic risk of the asset.

**Exhibit A1.1: Definition of Common Factors in Northfield U.S. Equity Fundamental Model**

SMART BETA FACTOR	NORTHFIELD RISK FACTORS	RISK FACTOR TYPE
Market Risk	Market Beta	Long Only
	Earnings-to-Price	Long-Short
Value	Book-to-Price	Long-Short
	Revenue-to-Price	Long-Short
Dividend Yield	Dividend Yield	Long-Short
Turnover	Trading Activity	Long-Short
Momentum	12-Month Relative Strength	Long-Short
Size	Logarithm of Market Cap	Long-Short
Earnings Stability	Earnings Variability	Long-Short
Earnings Growth	EPS Growth Rate	Long-Short
Debt	Debt-to-Equity	Long-Short
Price Volatility	Price Volatility	Long-Short
Sectors and Industry Groups	Industry	Dummy

Source: S&P Dow Jones Indices LLC and Northfield Information Services, Inc. Table is provided for illustrative purposes.

<sup>14</sup> Northfield Information Services, Inc. [Northfield U.S. Equity Fundamental Model](#).

<sup>15</sup> Exposures are standardized values of continuous variables (such as dividend yield) and of dummy variables (such as industry membership).

## APPENDIX 2<sup>16</sup>

Aked and Moroz propose a linear framework in which implicit implementation costs<sup>17</sup> from rebalancing rules-based portfolios can be estimated. Based on some simplifying assumptions, the implicit cost of rebalancing, expressed as a percentage of assets-under-management, is a function of five factors:<sup>18</sup> amount of assets invested, effective portfolio turnover, illiquidity tilt, stock turnover, and weighted average market cap of the strategy. It can be expressed with the following equation.

$$Cost = \frac{k A \times ETO}{\tau WAMC} (HI - LI)$$

where:

- $k$  is a **constant** that depends on the individual market and refers to the ratio of the change in a stock's pre-trade and post-trade prices with respect to its pre-trade price, compared with the ratio of the size of trade in U.S. dollars with respect to its aggregate volume across all the company's share classes in U.S. dollars. In this research paper,  $k$  is assumed to be 1 in the computation of our cost estimates.
- $A$  is the **amount of assets invested** in the index strategy.
- $ETO$  is the index strategy's **effective portfolio turnover** and is defined as the sum of replacement turnover and the square of adjusted weighting turnover. Replacement turnover represents the turnover generated by additions and deletions, while the adjusted weighting turnover represents the turnover derived from reweighting existing constituents. Turnover from reweighting is generally less expensive than additions and deletions, and this is recognized by taking the square of adjusted weighting turnover in the computation of effective turnover.
- $HI - LI$  is the index strategy's **illiquidity tilt**.  $HI$  is the Herfindahl index and  $LI$  is a measure of the liquidity of the portfolio, which indicates how far the portfolio deviates from the volume-weighted index. A volume-weighted index has the lowest implementation cost because it essentially makes use of all available volume to the largest extent.
- $\tau$  is the **stock turnover**, which is the ratio of traded volume to market cap of the index strategy.
- $WAMC$  is the **weighted average market cap of the strategy**, as defined in this research paper.

<sup>16</sup> For further details, please refer to Aked and Moroz (2015).

<sup>17</sup> The paper categorizes the implementation costs of an index-based strategy into explicit and implicit costs. Implicit costs relate to the unobserved reduction in the performance of an index on account of trading activity, whereas explicit costs refer to the difference between the performance of the fund and that of the index.

<sup>18</sup> In Aked and Moroz (2015), the five factors that determine the implicit cost of rebalancing are described slightly differently. This does not affect the mathematical outcome of the cost estimate computation.

**Exhibit A2.1: Further Comparative Statistics of Different Rankings- and Optimization-Based Diversified Value Portfolios**

CONSTRUCTION AND ANALYTICS		RANKING-BASED PORTFOLIO CONSTRUCTION			OPTIMIZATION-BASED PORTFOLIO CONSTRUCTION				
		PORTFOLIO 1	PORTFOLIO 2	PORTFOLIO 3	PORTFOLIO 4	PORTFOLIO 5	PORTFOLIO 6	PORTFOLIO 7	
Portfolio Construction	All constituents in strategy?	Yes	Yes	Yes	No	No	No	No	
	Weight Scheme	Market-cap adjusted value z-scores	Market-cap adjusted value z-scores	Market-cap adjusted value z-scores	Value z-scores	Optimized based on value z-scores	Optimized based on value z-scores	Optimized based on value z-scores	Optimized based on value z-scores
	Maximum Weight	4 times benchmark weight	4 times benchmark weight	4 times benchmark weight	4 times benchmark weight	4 times benchmark weight	4 times benchmark weight	15 times benchmark weight	
	Other constraints	-	Sector neutrality <sup>g</sup>	-	Sector neutrality <sup>b</sup>	Unintended factors neutrality <sup>c</sup>	Sector <sup>b</sup> and unintended factors neutrality <sup>c</sup>	Sector <sup>b</sup> and unintended factors neutrality <sup>c</sup>	
Portfolio Features	Maximum Number of Days to Trade (Days) <sup>c</sup>	1.444	1.702	3.987	3.459	2.041	2.295	4.809	
	Maximum Stock Ownership (%) <sup>c</sup>	0.161	0.206	0.400	0.316	0.140	0.219	0.376	
	Average Number of Constituents	500	500	500	354	323	323	318	
	Maximum Weight Multiplier (With Regard to S&P 500)	3.590	3.826	4.000	3.892	3.929	4.000	13.553	
Sector	Consumer Discretionary (%)	-0.964	-0.061	4.097	0.001	-1.084	0.040	0.018	
	Consumer Staples (%)	-0.599	0.083	-2.652	0.130	-0.895	0.136	0.141	
	Energy (%)	1.814	-0.623	-1.037	-0.450	1.875	-0.412	-0.364	
	Financials (%)	6.364	1.123	3.672	1.610	4.205	1.791	1.721	
	Health Care (%)	-1.801	-0.031	-3.293	-0.102	-2.045	-0.121	-0.110	
	Industrials (%)	-0.610	0.028	0.703	0.423	-0.140	0.359	0.315	
	Information Technology (%)	-4.292	0.418	-7.300	0.069	-3.024	0.064	0.045	
	Materials (%)	-0.145	0.048	2.477	-0.017	-0.578	-0.005	0.020	
	Real Estate (%)	-0.747	-1.107	0.369	-1.782	-1.831	-2.006	-1.917	
	Telecommunication Services (%)	0.213	0.078	-1.321	0.086	0.029	0.096	0.087	
	Utilities (%)	0.767	0.044	4.285	0.032	3.489	0.058	0.044	
Risk Analysis	Momentum Risk (% Tracking Error)	11.0	13.5	5.7	14.5	5.3	5.0	4.6	
	Volatility Risk (% Tracking Error)	1.8	2.1	1.9	2.1	1.2	0.8	0.9	
	Size Risk (% Tracking Error)	-0.2	-0.1	23.9	0.5	0.0	0.2	0.3	
	Beta Risk (% Tracking Error)	23.6	18.5	26.8	17.3	11.3	14.7	13.3	

Portfolios 1-7 are hypothetical portfolios.

Source: S&P Dow Jones Indices LLC and Northfield U.S. Fundamental Equity Model. Data from Dec. 31, 2009, to April 31, 2016. Past performance is no guarantee of future results. Chart is provided for illustrative purposes. Results for optimized portfolios will vary for risk parameters and optimization engines. Cells shaded light teal represent the highest number in that row and cells shaded light grey represent the lowest number. NOTE: <sup>a</sup> value risk is the sum of active risk from price-to-earnings, price-to-book, and price-to-sales; <sup>b</sup> sector neutrality means that all sector weights must be the same as the S&P 500 at rebalance; <sup>c</sup> factors neutrality means that all fundamental factors have to be within 0.05 standard deviations relative to the S&P 500; <sup>d</sup> calculated based on a floating portfolio size of 0.025% of total S&P 500 market value (or approximately USD 3.58 billion).

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