Commodity Index Mathematics
Methodology

January 2024

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Introduction

Overview

This document covers the mathematics of commodity index calculations and assumes some acquaintance with mathematical notation and simple operations. The calculations are presented principally as equations, which have largely been excluded from the individual index methodologies, with examples or tables of results to demonstrate the calculations.

Different Varieties of Indices

S&P Dow Jones Indices offers a variety of index types, calculated according to various methodologies, all of which are covered in this document. The common types of indices are, but are not limited to:

- **Production Weighted Indices**:

    o **Production Weighted Indices:** where constituent weights are determined by commodity world production quantities calculated as contract production weights (CPWs).

    o **Capped Indices:** where single commodity constituents or defined groups of commodity constituents (components), such as sector groups, are confined to a maximum index weight.

- **Non-Production Weighted Indices**:

    o **Liquidity Weighted Indices:** where constituent weights are determined by total dollar value traded (TDVT) calculated as contract weight factors (CWFs).

    o **Price Weighted Indices:** where constituent weights and index levels are determined solely by the commodity settlement prices in the index.

    o **Long-Short Indices:** where each commodity can have a long position or short position in the index.

- **Derived Indices**:

    o **Leveraged and Inverse Indices:** which return positive or negative multiples of their respective underlying indices.

    o **Weighted Return Indices:** commonly known as an "index of indices", where each underlying index is a component with an assigned weight to calculate the overall index of indices level.

    o **Other Derived Indices:** excess return, total return, risk control, decrement, currency, and currency hedged indices.

Supporting Documents

This methodology is meant to be read in conjunction with supporting documents providing greater detail with respect to the policies, procedures and calculations described herein. References throughout the methodology direct the reader to the relevant supporting document for further information on a specific topic. The list of the main supplemental documents for this methodology and the hyperlinks to those documents is as follows:

Supporting Document	URL
S&P Dow Jones Indices' Commodities Indices Policies & Practices Methodology	Commodities Indices Policies & Practices

This methodology was created by S&P Dow Jones Indices to achieve the aforementioned objective of measuring the underlying interest of each index governed by this methodology document. Any changes to or deviations from this methodology are made in the sole judgment and discretion of S&P Dow Jones Indices so that the index continues to achieve its objective.

Production Weighted Indices

Definition

Most of S&P Dow Jones Indices' commodity indices are production weighted indices. Production weighted indices have their individual weights determined based on World Production statistics compiled by external sources. Examples include the S&P GSCI, sector and single commodity versions. In the discussion below most of the examples will refer to the S&P GSCI.

For information on the S&P GSCI, please see the S&P GSCI Methodology, available at www.spglobal.com/spdji/.

Calculation of Spot Indices

The formula to calculate the S&P GSCI Spot is:

$$S\&P\ GSCI_d = \frac{TDW_d}{NC}$$

where:

TDW_d = The Total Dollar Weight for the current index business day (see below for derivation)

NC = The Normalizing Constant (see below for derivation)

Total Dollar Weight Calculation on Non-Roll Days

The Total Dollar Weight (TDW) is the sum of the Dollar Weights of all futures contract months. The formula for calculating the Total Dollar Weight of the index on any index business day that does not occur during a Roll Period is the following:

$$TDW_d = \sum_c (CPW_d^c * DCRP_d^c)$$

where:

c = The Designated Contract

d = The index business day on which the calculation is made

CPW = The current Contract Production Weight (see below for derivation)

$DCRP$ = The Daily Contract Reference Price

Total Dollar Weight Calculation During a Roll Period

The formula for calculating the Total Dollar Weight of the index on any index business day that occurs during a Roll Period (other than a reconstitution Roll Period or any other Roll Period in which a re-weighting of an index is effected) is the following:

$$TDW_d = \sum_c CPW^c * (CRW1_d^c * DCRP1_d^c + CRW2_d^c * DCRP2_d^c)$$

where:

c = Each Designated Contract

d = The index business day on which the calculation is made

$CRW1$ = The Contract Roll Weight of the First Nearby Contract Expiration

$CRW2$ = The Contract Roll Weight of the Roll Contract Expiration

$DCRP$ = The Daily Contract Reference Price of each respective Contract Expiration

Total Dollar Weight Calculation During Any Reweighting Period

$$TDW_d = \left(\frac{NC_{new}}{NC_{old}} * \sum_c [CPW1^c * CRW1_d^c * DCRP1_d^c]\right) + \sum_c [CPW2^c * CRW2_d^c * DCRP2_d^c]$$

where:

c = Each Designated Contract

d = The index business day on which the calculation is made

$CRW1$ = The Contract Roll Weight of the First Nearby Contract Expiration

$CRW2$ = The Contract Roll Weight of the Roll Contract Expiration

$CPW1$ = The CPW of the First Nearby Contract Expiration

$CPW2$ = The new CPW of the Roll Contract Expiration

$DCRP$ = The Daily Contract Reference Price of each respective Contract Expiration

In order to reflect this roll into the new CPWs and Normalizing Constant, the formula for the Total Dollar Weight of the index requires the additional adjustments detailed above. Specifically, because the CPWs of the First Nearby Contract Expiration and the Roll Contract Expiration will be different, CPW1 and CPW2, as set forth above, must enter the calculation. In addition, the result of this calculation must be multiplied by the Total Dollar Weight Ratio, which reflects the change in the Total Dollar Weight resulting from the shift to new CPWs and, therefore, when multiplied by CRW1 and CRW2, rolls the S&P GSCI into the new CPWs and the new Normalizing Constant.

Calculation of the Normalizing Constant

The Normalizing Constant (NC) is a factor that ensures continuity of an index level. The NC calculates on the reference day prior to the reconstitution period (for the S&P GSCI, the reconstitution period is in January) and is applied during the index designated roll period. The NC is rounded to seven digits of precision. The formula for calculating the NC is the following:

$$NC_{new} = NC_{old} * TDWR$$

where:

NC_{old} = The Normalizing Constant for the prior reconstitution period

$TDWR$ = The Total Dollar Weight Ratio

Total Dollar Weight Ratio

The Total Dollar Weight Ratio is calculated according to the following:

$$TDWR = \frac{\sum_c (CPW_{new}^c * DCRP_d^c)}{\sum_c (CPW_{old}^c * DCRP_d^c)}$$

where:

c = The Designated Contract

d = The index business day on which the calculation is made

CPW_{new} = CPWs that take effect on the first day of the new reconstitution period

| CPW_{old} | = The CPWs for the prior reconstitution period |
| $DCRP$ | = The Daily Contract Reference Price |

Calculation of the Contract Production Weights

In calculating the Contract Production Weight (CPW) of each Designated Contract on a particular commodity, the World Production Average (WPA) of such commodity is allocated to those Designated Contracts that can best support liquidity. The final CPWs are rounded to seven digits of precision. The new CPWs are implemented during the January roll period. The formula for calculating the Contract Production Weights is as follows:

$$CPW_i = \frac{CF * Percentage\ TQT_i * WPA_i}{1{,}000{,}000}$$

where:

CF	= The Conversion Factor for each designated commodity (see *Appendix CF* the S&P GSCI Index Methodology)
$Percentage\ TQT_i$	= Percentage of Total Quantity Traded for each designated commodity
WPA_l	= World Production Average for each designated commodity

CPW Adjustment Procedure

If the calculation of the CPWs for the Designated Contracts on a particular commodity results in the TVM of such Contracts being below the TVM Reweighting Level, then the CPWs for all such Contracts are reduced until the TVM of such Contracts is equal to the TVM Reweighting Level. The TVM Reweighting Level is set to 50. The formula for calculating the TVM is as follows:

$$TVM_c = \frac{TQT_c \sum_k (CPW_k * ACRP_k)}{ISL * CPW_c}$$

The adjustment procedure is designed to ensure that the CPW of each Designated Contract is at a level sufficient to support trading activity in the index, but not disproportionately high. This is achieved by setting the TVM for each such Contract at the TVM Reweighting Level and reducing the CPW for such Contract accordingly. The following procedure is used to adjust the CPWs of Designated Contracts, under the circumstances described above:

1. Determine the Designated Contracts to be re-weighted.

2. Calculate the CPWs for all Designated Contracts in according to the following formula:

$$CPW_{rl} = \frac{\dfrac{CS_c * V_c * \sum_k (CPW_k * ACRP_k)}{ISL * 1{,}000{,}000{,}000}}{TVM_{rl}}$$

where:

CS_c	= The Contract Size for each designated commodity
V_c	= The Total Aggregate Volume for each designated commodity
$ACRP_k$	= The Average Contract Reference Price for each designated commodity. The average is calculated using the current roll out contract price from the last business day of the designated reweighting period.
ISL	= Investment Support Level
TVM_{rl}	= Trade Volume Multiple Reweighting Level (set to 50)

3. If the CPW recalculation results in the breach of the TVM Reweighting Level, repeat step 2 until no further adjustment is necessary.

Capped Indices

Definition

A capped index is an index in which single commodity index constituents, or defined groups of commodity index constituents, are constrained to a maximum weight with any excess weight proportionally distributed among the remaining uncapped constituents. The standard capping rule, applied to most capped indices, is as follows:

At each rebalancing, if the largest commodity or component's weight exceeds 32% of the total index weight, the weight is capped at the target weight of 32%. Any other commodity or component with a weight exceeding 17% is capped at the target weight of 17%. As prices change, the weights shift, and the modified weights change. Therefore, a capped index must be rebalanced to re-establish the proper weighting. The capped indices use the same calculation methodology as production weighted indices.

For more information on the specific formulas used, please refer to the Production Weighted Indices section of this document.

There are two main examples of capped indices: capped commodity and capped component. This section focuses on the formulas necessary for calculating the target weights and the CPWs for both examples.

Capped Contract Production Weight Calculation

The Contract Production Weights (CPWs) calculated for the capped indices consider the underlying headline index CPW to determine the target weights used in the CPW calculation. The same CPW calculation applies to both capped component and capped commodity versions. At each rebalancing, CPWs are calculated as follows:

$$CPW_{capped\ i} = CPW_{index\ i} * \frac{TargetWeight_j}{IndexWeight_j}$$

where:

$CPW_{capped\ i}$	= CPW for commodity i in the capped index as of the upcoming rebalancing
$CPW_{index\ i}$	= CPW for commodity i in the headline index as of the upcoming rebalancing
$TargetWeight_j$	= Weight of commodity i in the capped index as of the upcoming rebalancing
$IndexWeight_j$	= Weight of commodity i in the headline index as of the upcoming rebalancing

Capped Commodity Target Weight Calculation

Capped commodity target weights are determined using the individual headline index weights at the time of rebalancing.

For information on capped commodity target weight rules, please refer to S&P GSCI Capped Indices Methodology.

At each rebalancing, target weights are calculated as follows:

If *IndexWeight $_i$* > 32%, then *TargetWeight $_i$* = 32%

For all remaining commodities in that sector:

$$TargetWeight_j = (SectorWeight_i - 32\%) * \frac{IndexWeight_j}{(SectorWeight_i - IndexWeight_c)}$$

where:

SectorWeight i = Weight of the sector in the headline index as of the upcoming rebalancing

IndexWeight c = Total weight of the capped commodities in sector c in the headline index as of the upcoming rebalancing

For any subsequent commodities:

If *IndexWeight i* > 17% then *TargetWeight i* = 17%

For all remaining commodities in that sector:

$$TargetWeight_j = (SectorWeight_i - 17\%) * \frac{IndexWeight_j}{(SectorWeight_i - IndexWeight_c)}$$

where:

SectorWeight i = Weight of the sector in the headline index as of the upcoming rebalancing

IndexWeight c = Total weight of the capped commodities in sector c in the headline index as of the upcoming rebalancing

This process repeats iteratively until no more than one commodity's weight exceeds 17% of the total index weight.

Thus, for any uncapped commodities in a sector which has more than one capped commodity, weights calculate as follows:

$$TargetWeight_i = (SectorWeight_i - CappedWeight) * \frac{IndexWeight_i}{(SectorWeight_i - IndexWeight_c)}$$

where:

CappedWeight = Total weight of all capped commodities for the sector in question

If the final commodity in a given sector has a weight greater than 17% after the prior iteration, the weight of that commodity is capped at 17%, with the excess weight proportionately redistributed among the remaining uncapped commodities in the index.

Capped Component Target Weight Calculation

Capped component target weights are determined using the individual headline weights at the time of rebalancing, along with the components.

For information on capped component target weight rules, please refer to S&P GSCI Capped Indices Methodology.

At each rebalancing target weights are calculated as follows:

If *IndexWeight j* > 32%, then *TargetWeight j* = 32%

For all remaining commodities in that sector:

$$TargetWeight_j = \frac{(68\% * IndexWeight_j)}{(100\% - IndexWeight)}$$

where:

    IndexWeight = Total weight of the capped components in the headline index as of the upcoming rebalancing

For any subsequent commodities:

    If *IndexWeight $_j$* > 17% then *TargetWeight $_j$* = 17%

For all remaining commodities in that sector:

$$TargetWeight_{\,j} = \frac{(100\% * CappedWeight) * IndexWeight_{\,j}}{(100\% - IndexWeight)}$$

where:

    IndexWeight = Total weight of the capped components in the headline index as of the upcoming rebalancing.

This process repeats iteratively until no more than one commodity's weight exceeds 17% of the total index weight.

Single Commodity Capped Component Index Calculation

Single commodity capped component indices cap the individual commodity weight at 32% with the remaining weight being equally distributed among the rest of the commodity eligible in the index. Additional versions of single capped indices are also available.

For information on single capped component rules and target weights, please refer to S&P GSCI Capped Indices Methodology.

Spot & Excess Return Index Calculation

$$Index_t = Index_{t_R} * \sum_{i=1}^{N} \left(Weight_{\,i\,t_R} * \frac{SingleIndex_{\,i\,t}}{SingleIndex_{\,i\,t_R}} \right)$$

where:

Index $_t$	= Index Capped Component index level
t_R	= Rebalancing date
Weight $_I$	= Monthly weight reset
SingleIndex $_t$	= Underlying single commodity index level for the *i^{th}* component
N	= Total number of components in the index

Total Return Index Calculation

$$TR_t = TR_{t-1} * \left(\frac{ER_t}{ER_{t-1}} + TBR_t \right) * (1 + TBR_t)^{days}$$

where:

    TR $_t$ = Total Return Index Capped Component index level

    days = Number of non-index business days between *T* and *T $_{t-1}$*

$$TBR_t = \left[\frac{1}{1 - \frac{91}{360} * TBAR_{t-1}}\right]^{\frac{1}{91}} - 1$$

where:

  $TBAR_{t-1}$ = 91-day discount rate for U.S. Treasury Bills, as reported by the U.S. Department of the Treasury's Treasury Direct service *here*.

Liquidity Weighted Indices

Definition

A liquidity weighted index is one in which index weights are determined by the liquidity of the underlying commodities. Examples include the Dow Jones Commodity Index (DJCI), and sector and single commodity versions, with most of the examples referring to the DJCI.

Liquidity is measured in commodity indices through Total Dollar Value Traded (TDVT), which is a measure of the dollar amount of liquidity using commodity settlement prices, volume, and the commodity contract size. TDVT is calculated annually, with a calculation period of the prior year September through August of the current year. Liquidity weighted indices generally rebalance annually to allow for the research and compilation of the necessary inputs to calculate TDVT.

The calculation for TDVT is as follows:

$$TDVT_c = CS_c * V_c * ACRP_k$$

where:

CS_c = The Contract Size for each designated commodity

V_c = The Total Aggregate Volume for each designated commodity

$ACRP_k$ = The Average Contract Reference Price for each designated commodity

Calculation of the Normalizing Constant

The Normalizing Constant is typically calculated on the designated index business day prior to the rebalance period (for the Dow Jones Commodity Index, the rebalance is monthly, and the reconstitution period is in January) and is applied during the index designated roll period. The Normalizing Constant is rounded to seven digits of precision. The formula for calculating the Normalizing Constant is the following:

$$NC_{new} = NC_{old} * TDWR$$

where:

NC_{old} = The Normalizing Constant for the prior reconstitution period

$TDWR$ = The Total Dollar Weight Ratio

Total Dollar Weight Ratio

The Total Dollar Weight Ratio is calculated according to the following:

$$TDWR = \frac{\sum_c (CWF_{new}^c * DCRP_d^c)}{\sum_c (CWF_{old}^c * DCRP_d^c)}$$

where:

c = The Designated Contract

d = The index business day on which the calculation is made

CWF_{new} = Contract Weight Factors that take effect on the first day of the rebalance

CWF_{old} = The Contract Weight Factors prior rebalance period

DCRP = The Daily Contract Reference Price

Commodity Weight Calculation

The individual commodity weights are determined based on the overall Total Dollar Value Traded calculation. These weights are used to calculate the Contract Weight Factor (CWF) for each commodity. The calculation for the individual commodity weights is as follows:

$$CW_c = \frac{TDVT_N}{\sum_N (TDVT_N)}$$

where:

N = The total number of commodities in the composition of the index

Commodity Weight Factor Calculation

The individual CWFs are calculated during each designated rebalance period. The CWFs are determined based on the final individual commodity weights. The final weights are based on any weight adjustments determined from the designated liquidity weighted index methodology. The individual CWFs are calculated as follows:

$$CWF_i = \frac{FinalWeight_c}{\left(\frac{Price_N}{\sum_N (Price_N)}\right)}$$

where:

CWF_I	= The CWF for commodity i in the index as of the rebalancing reference date
$Price_N$	= The price for commodity N in the index as of the rebalancing reference date
$FinalWeight_c$	= The weight of commodity i, in the index as of the rebalancing reference date

Price Weighted Indices

Definition

In a price weighted index, constituent weights and index levels are determined exclusively by the daily futures contract settlement prices. Price weighted indices are single commodity indices that typically do not rebalance. Some examples of price weighted indices are the Equity Futures & Currency Futures index families, as well as the S&P Global Bond futures index family.

Calculation of the Excess Return Indices

The excess return of each of the indices is calculated from the price change of the underlying futures contract. On any trading date, t, the level of each of the sub-indices is calculated as follows:

$$IndexER_d = IndexER_{d-1} * (1 + CDR_d)$$

where:

  IndexER $_{d-1}$ = The Excess Return Index level on the preceding business day, defined as any date on which the index is calculated

  CDR $_d$ = The Contract Daily return, defined as:

$$CDR_d = \frac{TDWO_t}{TDWI_{t-1}} - 1$$

   where:

    $TDWO_t = CRW1_{t-1} * DCRP1_t + CRW2_{t-1} * DCRP2_t$

    $TDWI_{t-1} = CRW1_{t-1} * DCRP1_{t-1} + CRW2_{t-1} * DCRP2_{t-1}$

    t   = The business day on which the calculation is made

    CRW1 = The contract roll weight of the first nearby contract expiration

    CRW2 = The contract roll weight of the roll in contract expiration

    DCRP $_t$ = The Daily Contract Reference Price of the futures contract

Long-Short Indices

Definition

Long-Short Indices measure the performance of a strategy based on trends in price movements of specific futures contracts. Futures contracts are represented individually on either a long or short basis, depending on market momentum. Long or short positions are determined by measuring the current futures contract prices relative to an exponential moving average. The moving average is determined by the individual commodity index methodology.

Calculation of the Normalizing Constant

The Normalizing Constant is typically calculated on the designated index business day prior to the rebalance period (for long-short indices, the rebalance occurs monthly) and is applied during the index designated roll period. The Normalizing Constant is rounded to seven digits of precision. The formula for calculating the Normalizing Constant is the following:

$$NC_{new} = NC_{old} * \frac{\sum(CPW_{new} * DCRP1_d + CPW_{new} * DCRP2_d) + SC1}{\sum(CPW_{old} * DCRP1_d + CPW_{old} * DCRP2_d) + SC2}$$

where:

CPW_{new}	=	The CPW for the current month
CPW_{old}	=	The CPW for the previous month
$SC1$	=	The Short Component effective during the last month expressed as a CPW
$SC2$	=	The Short Component effective in the current month expressed as a CPW
$DCRP1_d$	=	The current contract reference price on day d
$DCRP2_d$	=	The next contract reference price on day d
NC_{old}	=	The Normalizing Constant effective during the last month

Calculation of Spot Indices

Long-short indices consider a short component in the calculation of the index. The Short Component *(SC)* is allocated to the amount of weight remaining in the Index after the weights of each component has been defined based on the long and short positions and their respective percentage weights. Adding the weight of the Short Component to the sum of the weights of the Components will make the weights in the Index sum to 100%. The Short Component is calculated as follows:

$$SC = \left(1 - \sum ComponentWeights\right) * 1000$$

On a given business day, *d*, the spot index level is calculated as follows:

$$SPOT_d = \frac{(\sum_{c=1}^{i} TDW1 + SC1)}{NC_{old}} + \frac{(\sum_{c=1}^{i} TDW2 + SC2)}{NC_{new}}$$

where:

$\sum_{c=1}^{i} TDW1$	=	The sum of the Total Dollar Weight of each components' current contract
$\sum_{c=1}^{i} TDW2$	=	The sum of the Total Dollar Weight of each components' next contract
$SC1$	=	The Short Component effective during the last month expressed as a CPW
$SC2$	=	The Short Component effective in the current month expressed as a CPW

NC_{old} = The Normalizing Constant effective during the last month

NC_{new} = The Normalizing Constant effective during the current month

Total Dollar Weight Calculation

On any day, d, the Total Dollar Weight (TDW) for Commodity c is the product of its Contract Production Weight, Contract Roll Weight and Daily Contract Price for the current and next contracts, respectively.

$$TDW_d = CPW^c * CRW_d^c * DCRP_d^c$$

where:

c = Each Designated Contract

d = The index business day on which the calculation is made

CPW_c = The CPW of the First Nearby Contract Expiration set on the start of the rebalancing period

$CRW^c{}_d$ = The Contract Roll Weight of the First Nearby Contract Expiration

$DCRP^c{}_d$ = The Daily Contract Reference Price of each respective Contract Expiration

Contract Production Weight Calculation

CPWs are determined the business day prior to the designated rebalance month and are implemented over the designated roll period, as per the index methodology. CPW values are calculated as follows:

$$CPW = \frac{ComponentWeight}{DCRP_d * 1000}$$

Calculation of Excess Return Indices

The Excess return index level is calculated using the previous days' index level and one plus the Contract Daily Return. Depending on the individual index methodology, the index levels used in the calculation are either using the full decimal precision or are rounded to seven digits of precision. The index calculation is as follows:

$$IndexER_d = IndexER_{d-1} * (1 + CDR_d)$$

Calculation of the Contract Daily Return

The Contract Daily Return is equal to the ratio of the Total Dollar Weight Obtained (TDWO) on such Day and the Total Dollar Weight Invested (TDWI) on the preceding business day, minus one. TDWI is the previous days' Total Dollar Weight (TDW). TDWO is calculated in the same manner as the TDW but uses the previous day's CPWs and contract roll weights, along with the current day's Daily Contract Reference Prices.

$$CDR_d = \frac{TDWO_d}{TDWI_{d-1}} - 1$$

The Total Dollar Weight Obtained (TDWO) calculation is as follows:

$$TDWO_d = \frac{NC_{new}}{NC_{old}} * \left[\left(\sum^i (CPW1_{d-1} * CRW1_{d-1} * DCRP1_d) + SC1 * CRW1_{d-1} \right) + \left(\sum^i (CPW2_{d-1} * CRW2_{d-1} * DCRP2_d) + SC2 * CRW2_{d-1} \right) \right]$$

The Total Dollar Weight Invested (TDWI) calculation is as follows:

$$TDWI_d = \frac{NC_{new}}{NC_{old}} * \left[\left(\sum^i (CPW1_{d-1} * CRW1_{d-1} * DCRP1_{d-1}) + SC1 * CRW1_{d-1} \right) + \left(\sum^i (CPW2_{d-1} * CRW2_{d-1} * DCRP2_{d-1}) + SC2 * CRW2_{d-1} \right) \right]$$

Futures-Based Leveraged & Inverse Indices

Definition

Futures-based Leveraged Indices are designed to generate a multiple of the return of the underlying futures index in situations where the investor borrows funds to generate index exposure beyond their cash position.

Futures-based Inverse indices are designed to provide the inverse performance of the underlying futures index; this represents a short position in the underlying index. The approach is to first calculate the underlying index, then calculate the daily returns for the leveraged or inverse index. There is no change to the calculation of the underlying futures index.

The leveraged or inverse index may rebalance daily or periodically.

Daily Rebalanced Leverage or Inverse Futures Indices

If the S&P Dow Jones Indices futures-based leveraged or inverse index is rebalanced daily, the index excess return is the multiple of the underlying index's excess return and calculated as follows:

$$IndexER_t = IndexER_{t-1} * \left(1 + \left(K * \left(\frac{UnderlyingIndexER_t}{UnderlyingIndexER_{t-1}} - 1 \right) \right) \right)$$

where:

K ($K \neq 0$) = Leverage/Inverse Ratio

- $K = 1$, no leverage
- $K = 2$, leverage exposure = 200%
- $K = 3$, leverage exposure = 300%
- $K = -1$, inverse exposure = -100%

A total return version of each of the indices is calculated, which includes interest accrual on the notional value of the index based on a specified interest rate (e.g., 91-day U.S. Treasury rate), as follows:

$$IndexTR_t = IndexTR_{t-1} * \left(\left(\frac{IndexER_t}{IndexER_{t-1}} \right) + TBR_t \right)$$

where:

$IndexTR_{t-1}$ = The Index Total Return on the preceding business day

TBR_t = Treasury Bill Return, as determined by the following formula:

$$TBR_t = \left[\frac{1}{1 - \frac{91}{360} * TBAR_{t-1}} \right]^{\frac{Delta_t}{91}} - 1$$

$Delta_t$ = The number of calendar days between the current and previous business days

$TBAR_{t-1}$ = The most recent weekly high discount rate for the 91-day U.S. Treasury bills effective on the prior business day.

Periodically Rebalanced Leverage or Inverse Futures Indices

If the futures-based leveraged or inverse index rebalances periodically (e.g., weekly, monthly, or quarterly), the index excess return is the multiple of the underlying index excess return since last rebalancing business day and shall be calculated as follows:

$$IndexER_t = IndexER_{t\_LR} * \left(1 + \left(K * \left(\frac{UnderlyingIndexER_t}{UnderlyingIndexER_{t\_LR}} - 1 \right) \right) \right)$$

where:

$IndexER_{t\_LR}$	= The Index Excess Return on the last rebalancing business day, t_LR
$UnderlyingIndexER_{t\_LR}$	= The Underlying Index Excess Return value on the last rebalancing business day, t_LR
t_LR	= The last rebalancing business day

K (K ≠ 0) = Leverage / Inverse Ratio

- K = 1, no leverage
- K = 2, leverage exposure = 200%
- K = 3, leverage exposure = 300%
- K = -1, inverse exposure = -100%

A total return version of each of the indices is calculated, which includes interest accrual on the notional value of the index based on the 91-day U.S. Treasury rate. The formulae are the same as above.

Weighted Return Indices

Definition

Weighted Return Indices combine the returns of two or more component indices using a specified set of weighting rules to create a new unique index return series. An index that uses the Weighted Return methodology may also be referred to as an "Index of Indices." Weighted Return indices may include a cash component which for the purposes of these indices is treated as a component index. S&P Dow Jones Indices offers both daily and periodic rebalancing approaches for weighted return indices.

Daily Rebalanced Weighted Return Futures Indices

If the S&P Dow Jones Indices futures-based weighted return index is rebalanced daily, the index excess return is the multiple of the underlying index's excess return and calculated as follows:

$$IndexER_t = IndexER_{t-1} * \left(1 + \sum_{i=1}^{N} \left(Weight_{i,t} * \left(\frac{UnderlyingIndexER_t}{UnderlyingIndexER_{t-1}} - 1\right)\right)\right)$$

where:

$Weight_{i,t}$	=	The weight of the underlying excess return index on day t
N	=	The number of component indices within the top-level index

Periodically Rebalanced Weighted Return Futures Indices

If the futures-based weighted return index rebalances periodically (e.g., weekly, monthly, or quarterly), the index excess return is the multiple of the component index excess return since the last rebalancing business day and is calculated as follows:

$$IndexER_t = IndexER_{t\_LR} * \left(1 + \sum_{i=1}^{N} \left(Weight_{i,r} * \left(\frac{UnderlyingIndexER_t}{UnderlyingIndexER_{t\_LR}} - 1\right)\right)\right)$$

where:

$IndexER_{t\_LR}$	=	The Index Excess Return on the last rebalancing business day, t_LR
$UnderlyingIndexER_{t\_LR}$	=	The underlying Index Excess Return value on the last rebalancing business day, t_LR
t_LR	=	The last rebalancing business day
$Weight_{i,r}$	=	The weight of the underlying excess return index on rebalancing date r
N	=	The number of component indices within the top-level index

Total return versions of each of the indices are calculated, which includes interest accrual on the notional value of the index based on a specified interest rate (e.g., 91-day U.S. Treasury rate). The formula is the same as provided in the *Leveraged & Inverse Indices* section.

Factor Indices Calculation

Factor Indices are composed of a long sub-index and a short sub-index. The Long Sub-Index is composed of the long front futures contract. The Short Sub-Index is composed of the short front futures contract. Each Index is calculated to reflect the corresponding inter-commodity spread, which is the difference in the daily changes (positive or negative) between the value of the Long Sub-Index and the value of the Short Sub-Index. The excess return calculation is as follows:

$$IndexER_t = IndexER_{t-1} * \left(1 + \left(W_l * \frac{LongIndexER_t}{LongIndexER_{t-1}} - 1\right) + \left(W_s * \frac{ShortIndexER_t}{ShortIndexER_{t-1}} - 1\right)\right)$$

where:

$IndexER_{t-1}$	=	The Index Excess Return on the preceding business day
W_l	=	The weight of the long index component
$LongIndexER_t$	=	The long index component on date t
W_s	=	The weight of the short index component
$ShortIndexER_t$	=	The short index component on date t

Total return versions of each of the indices are calculated, which includes interest accrual on the notional value of the index based on a specified interest rate (e.g., 91-day U.S. Treasury rate). The formula is the same as provided in the *Leveraged & Inverse Indices* section.

Other Derived Indices

Excess Return Indices

Excess return indices represent the return of a portfolio of commodity futures contracts and rolling them forward each month during a designated roll period, which generally keeps investment in the nearby futures contract months. The indices calculate based on the Contract Daily Return.

Calculation of Excess Return Indices

The Excess return index level is calculated using the previous day's index level and one plus the Contract Daily Return. Depending on the individual index methodology, the index levels used in the calculation are either using the full decimal precision or are rounded to seven digits of precision. The index calculation is as follows:

$$IndexER_d = IndexER_{d-1} * (1 + CDR_d)$$

Calculation of the Contract Daily Return

The Contract Daily Return is equal to the ratio of the Total Dollar Weight Obtained (TDWO) on such Day and the Total Dollar Weight Invested (TDWI) on the preceding business day, minus one. TDWI is the previous days' Total Dollar Weight (TDW). TDWO is calculated the same way as the TDW but uses the previous days' CPWs and contract roll weights, along with the current days' Daily Contract Reference Prices.

$$CDR_d = \frac{TDWO_d}{TDWI_{d-1}} - 1$$

Total Return Index Calculation Using Actual Day Counts

Total return indices incorporate the returns from the Excess Return indices and interest earned on hypothetical fully collateralized contract positions (the interest rate of return, or TBR) on the commodities included in the indices. The TBR assumes a 360-day calendar year, uses an actual day count, and also uses the 91-day Treasury Bill Rate, unless otherwise specified in the index methodology. Depending on the individual index methodology, the index levels used in the calculation are either using the full decimal precision or are rounded to seven digits of precision. The index calculation is as follows:

$$IndexTR_d = IndexTR_{d-1} * (1 + CDR_d + TBR_d) * (1 + TBR_d)^{days}$$

where:

$IndexTR_d$	= The Index Total Return on the preceding business day
$days$	= The number of business days since the preceding business day
TBR_d	= Interest rate of Return, as determined by the following formula:

$$TBR_d = \left[\frac{1}{1 - \frac{91}{360} * TBAR_{d-1}} \right]^{\frac{1}{91}} - 1$$

where:

$TBAR_{d-1}$ = 91-day Treasury Bill Rate available on the preceding business day.[1]

Total Return Index Calculation Using Business Day Counts

Some specific index methodologies call for using a business day count to calculate the interest rate of return instead of actual days. For a funded investment, the total return between dates t-1 and t includes the risk-free return for the initial cash outlay is calculated as follows:

$$IndexTR_d = IndexTR_{d-1} * IndexTotalReturn_d$$

where:

$$IndexTotalReturn_d = (1 + CDR_d + TBR_d)$$

$$TBR_d = \left[\frac{1}{1 - \frac{91}{360} * TBAR_{d-1}}\right]^{\frac{Delta_t}{91}} - 1$$

where:

$Delta_t$ = The number of calendar days between the current and previous business days

CDR_d = The Contract Daily Return

For information on Contract Daily Return calculations for Price Weighted indices, please refer to the 'Price Weighted Indices' section in the methodology.

Overnight Fed Fund Rate Total Return Index Calculation

Some specific index methodologies call for using the overnight Federal Funds Rate to calculate the interest rate of return. For a funded investment, the total return between dates t-1 and t is calculated as follows:

$$TBR_d = \left[\frac{1}{1 - \frac{91}{360} * FFR_{d-1}}\right]^{\frac{Delta_t}{91}} - 1$$

where:

$Delta_t$ = The number of calendar days between the current and previous business days

FFR_d = The simple Fed Funds rate, as determined by the following formula:

$$FFR_d = FF_{d-1} * \frac{Delta_d}{360}$$

where:

$Delta_t$ = The number of calendar days between the current and previous business days

FF_{d-1} = The most recent overnight federal funds rate, effective on the preceding business day

Futures-Based Risk Control Indices

When the underlying index is based on futures contracts, most of the Risk Control methodology follows the details in the *S&P Dow Jones Indices' Index Mathematics Methodology*. However, there are some differences, particularly as it relates to the cash component of the index.

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[1] The source is the U.S. Department of the Treasury's Treasury Direct service *here* on the most recent weekly auction date.

For such an index, it includes a leverage factor that changes based on realized historical volatility. If realized volatility exceeds the target level of volatility, the leverage factor will be less than one; if realized volatility is lower than the target level, the leverage factor may be greater than one. A given risk control index may have a maximum leverage factor that cannot be exceeded.

For equity risk control indices, the return consists of two components: (1) the return on the position in the underlying S&P Dow Jones Indices index and (2) the interest cost or gain, depending upon whether the position is leveraged or deleveraged. For futures-based risk control indices, there is no borrowing or lending to achieve investment objectives in the underlying index. Therefore, the cash component of the Index does not exist.

Again, a leverage factor greater than one represents a leveraged position, a leverage factor equal to one represents an unleveraged position, and a leverage factor less than one represents a deleveraged position. The leverage factor may change at regular intervals, in response to changes in realized historical volatility, or when the expected volatility exceeds or falls below predetermined volatility thresholds, if such thresholds were in place.

The formula for calculating the Risk Control Excess Return Index largely follows that detailed beginning with the equation detailed below. However, since there is no funding for such indices (as opposed to the case with equity excess return indices, where it is assumed, the initial investment is borrowed, and excess cash is invested), the interest rate used in the calculation is eliminated:

$$RC\ ER\ Index_t = (RC\ ER\ Index_{rb}) * (1 + RC\ ER\ IndexReturn_t)$$

where:

$$RC\ ER\ IndexReturn_t = K_{rb} * \left(\frac{Underlying\ Index_t}{Underlying\ Index_{rb}} - 1\right)$$

K_{rb} = The leverage factor set at the last rebalancing date, calculated as:

Min(Max K, Target Volatility/Realized Volatility $_{rb-d}$)

The formula for calculating the Risk Control Total Return Index, which includes interest earned on Treasury Bills, is as follows:

$$RC\ TR\ Index_t = (RC\ TR\ Index_{rb}) * (1 + RC\ TR\ IndexReturn_t)$$

where:

$$RC\ TR\ IndexReturn_t = \left[1 + \left[K_{rb} * \left(\frac{Underlying\ Index_t}{Underlying\ Index_{rb}} - 1\right) + \left[\prod_{i=rb+1}^{t}(1 + IR_{i-1} * \frac{D_{i-1,i}}{360}) - 1\right]\right]\right]$$

IR_{i-1} = The interest rate set for the index

Exponentially Weighted Volatility for Futures-Based Risk Control Indices

For futures-based risk control indices there is a three (3)-day lag between the calculation of the leverage factor based on the ratio of target volatility to realized volatility, and the implementation of that leverage factor in the index.

For information on Exponentially Weighted Volatility, please refer to the Risk Control 2.0 Indices section of S&P Dow Jones Indices' Index Mathematics Methodology.

Dynamic Volatility Risk Control Indices

In dynamic volatility risk control indices, the volatility target is not set as a definition of the index. Instead, the volatility target is set at various levels based on the moving average of VIX computed over a predetermined number of days (e.g., 30-day moving average).

Variance Based Risk Control Indices

In variance-based risk control indices, a target level of variance is set rather than a target volatility level. This allows for faster leveraging or deleveraging of allocations based on changes in volatility or variance in the market. For these indices:

$K_{rb} = Min(Max\ K, Target\ Variance/Realized\ Variance_{rb-d})$

where variance is defined as per above. All other index calculations remain the same.

For information on the additional formulae related to all Risk Control indices, please refer to the S&P Dow Jones Indices' Index Mathematics Methodology.

Currency & Currency Hedged Indices

Currency and Currency Hedged index calculations are standard across all asset classes.

For information and formulas on Currency and Currency Hedged index calculation rules, please refer to S&P Dow Jones Indices' Index Mathematics Methodology.

Disclaimer

Performance Disclosure/Back-Tested Data

Where applicable, S&P Dow Jones Indices and its index-related affiliates ("S&P DJI") defines various dates to assist our clients in providing transparency. The First Value Date is the first day for which there is a calculated value (either live or back-tested) for a given index. The Base Date is the date at which the index is set to a fixed value for calculation purposes. The Launch Date designates the date when the values of an index are first considered live: index values provided for any date or time period prior to the index's Launch Date are considered back-tested. S&P DJI defines the Launch Date as the date by which the values of an index are known to have been released to the public, for example via the company's public website or its data feed to external parties. For Dow Jones-branded indices introduced prior to May 31, 2013, the Launch Date (which prior to May 31, 2013, was termed "Date of introduction") is set at a date upon which no further changes were permitted to be made to the index methodology, but that may have been prior to the Index's public release date.

Please refer to the methodology for the Index for more details about the index, including the manner in which it is rebalanced, the timing of such rebalancing, criteria for additions and deletions, as well as all index calculations.

Information presented prior to an index's launch date is hypothetical back-tested performance, not actual performance, and is based on the index methodology in effect on the launch date. However, when creating back-tested history for periods of market anomalies or other periods that do not reflect the general current market environment, index methodology rules may be relaxed to capture a large enough universe of securities to simulate the target market the index is designed to measure or strategy the index is designed to capture. For example, market capitalization and liquidity thresholds may be reduced. In addition, forks have not been factored into the back-test data with respect to the S&P Cryptocurrency Indices. For the S&P Cryptocurrency Top 5 & 10 Equal Weight Indices, the custody element of the methodology was not considered; the back-test history is based on the index constituents that meet the custody element as of the Launch Date. Back-tested performance reflects application of an index methodology and selection of index constituents with the benefit of hindsight and knowledge of factors that may have positively affected its performance, cannot account for all financial risk that may affect results and may be considered to reflect survivor/look ahead bias. Actual returns may differ significantly from, and be lower than, back-tested returns. Past performance is not an indication or guarantee of future results.

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