Ossiam US Minimum Variance Index

Technical Description

March 29, 2016

1. Universe

1.1 Index Schedule

For the purpose of this document, a Business Day means any day for which the US equity market is open.

The index composition is reviewed monthly. The new composition is effective at the Effective Date (the first Business Day after the third Friday of the month: the "Rebalancing Date"). Should the Rebalancing Date be a non-Business Day, the Rebalancing Date shifts forward to the first Business Day after the third Friday. The Estimation Date occurs K Business Days before the Rebalancing Date.

The optimal weights are computed the first Business Day after the Estimation Date (i.e. K-1 Business Days before the Rebalancing Date) and drifted according to market prices for implementation after the close of US market on the Rebalancing Date.

1.2 Reference Universe

Investment universe for the Index is the forecasted composition of the S&P 500 index at the Effective Date. Therefore, it is the current (at the Estimation Date) composition of the S&P 500 to which scheduled Additions and Deletions between the Estimation Date and the Effective Date are applied.

1.3 Data

The following data is used in the Index construction process:

- P - daily share prices at market close,
- TR – daily share total return price. The total return price is adjusted for corporate actions and dividend payments,
- V - daily transaction volume from the US stock exchanges (NYSE, NASDAQ),
- C - London 4pm foreign exchange rates, as published by the WM Company
- classification of the stocks in S&P 500 by industrial sector corresponding to the level 1 of the GICS classification, having 10 industries:
    1. Energy
    2. Materials
    3. Industrials
    4. Consumer Discretionary
    5. Consumer Staples
    6. Health Care
    7. Financials
    8. Information Technology
    9. Telecommunication Services
    10. Utilities

1.4 Liquidity Filter

Only the most liquid stocks from the investment universe are selected. For this purpose a liquidity filter is designed, that works in the following way:

- liquidity is estimated for each stock, using most recent transaction volume data from primary exchange. Average Daily Volume is calculated as a simple average of daily transaction volume series over the past *Tv* days

$$ADV^i = \frac{1}{Tv} \sum_{t=T-Tv+1}^{T} V_t{}^i * P_t^i$$

where T denotes Estimation Date, V is volume in number of shares and P is stock price in the USD. Stocks with more than Tv*q% missing data are assigned zero ADV.

- the stocks from the investment universe are ranked by their ADV in descending order,
- first M stocks having the highest liquidity are selected.

The liquidity filter is applied each time the Index is rebalanced, before calculating new optimized weights.

1.5 Missing Data Filters

1.5.1 Volatility Estimation

The Eligible Universe consists of those stocks whose recent price history does not exhibit large non trading periods, which could bias the volatility estimation for this stock. For this purpose, a missing data filter is designed, performing the following actions: for each stock in the liquidity filtered selection, we count the number of Non Trading Occurrences (NNT) as follows

$$NNT_T^i = \frac{1}{T_s} \sum_{t=T-T_s+1}^{T} 1_{\{Stock\ has\ not\ close\ price\}}$$

where T denotes Estimation date and T_s is the number of days used for the volatility estimation. All stocks with NNT≥q% are discarded.

1.5.2 Correlation Estimation

The Eligible Universe consists of those stocks whose recent price history does not exhibit large non trading periods, which could bias the correlation estimation for this stock. For this purpose a missing data filter is designed, it performs the following actions: for each stock in the liquidity filtered selection, we count the number of Non Trading Occurrences (NNT) as follows

$$NNT_T^i = \frac{1}{T_r} \sum_{t=T-T_r+1}^{T} 1_{\{Stock\ has\ not\ close\ price\}}$$

where T denotes Estimation date and T_r is the number of days used for the correlation estimation.

All stocks with NNT≥q% are discarded

2 Portfolio Construction

2.1 General

Index constituents are weighted by an optimization procedure, aimed at minimizing portfolio variance under constraints.

2.2 Return Data

The optimization procedure starts by calculating daily arithmetic price returns:

$$r_t^i = \frac{TR_t^i}{TR_{t-1}^i} - 1$$

where (t-1) denotes the previous Business Day, TR are total return close prices.

2.3 Variance Estimation Details

For all the stocks admitted to the optimization step a variance-covariance matrix is estimated as follows:

$$\Sigma_{\mathrm{T}}^{\mathrm{i,j}} = \sigma_{\mathrm{T}}^{\mathrm{i}} * \sigma_{\mathrm{T}}^{\mathrm{j}} * \rho_{\mathrm{T}}^{\mathrm{ij}}$$

where M is the number of stocks admitted for optimization, σ^i is volatility of the i-th stock, and ρ^{ij} is correlation between the stocks (*i,j*). The ingredients of the covariance matrix are estimated on arithmetic daily returns as follows:

$$\sigma_T^i = \sqrt{\frac{1}{Ts-1}\sum_{t=T-Ts+1}^{T}(r_t^i - \bar{r}^i)^2}$$

where Ts is volatility estimation in days, T is the Estimation Date and $\bar{r}$ denotes simple average of stock returns. And correlation coefficients are estimated as

$$\rho_T^{i,j} = \frac{\frac{1}{Tr-1}\sum_{t=T-Tr+1}^{T}(r_t^i - \bar{r}^i)(r_t^j - \bar{r}^j)}{\sigma_T^i * \sigma_T^j}$$

where Tr is correlation estimation period in days, and volatilities in the denominator are estimated over the Tr-day period.

Only the days when all the stocks were actually traded are included in the estimation of the means and the variance-covariance (see more details in the section 2.8).

2.4 Optimization: objective function

The function to be minimized is the variance of the Index portfolio:

$$\sigma_{Ind}^2 = \sum_{i=1}^{M}\sum_{j=1}^{M} w_i \Sigma_{ij} w_j$$

2.5 Optimization: constraints

The optimization is subject to the following constraints:

- 100% leverage constraint: $\sum_{i=1}^{M} w_i = 1$
- long-only constraint: $w_i \geq 0$, for all *i*
- maximal weight constraint: $w_i \leq w_{max}$
- maximal sector exposure constraint: $w_S \leq S_{max}$

where $w_S = \sum_{i \in S} w_i$, is net exposure to the sector S.

- diversification target: $\sum_{i=1}^{M} w_i^2 = \frac{1}{H}$

2.6 Optimization: numerical algorithm

The optimization problem is a quadratic constrained minimization problem. It is solved numerically, using an interior-point algorithm. This algorithm calculates an iterative sequence of approximate minimization problems, where inequality constraints are transformed into equality constraints using slack variables. The optimal solution is defined with the help of the following convergence criteria:

- TolFun - termination tolerance on the function value,
- TolCon - tolerance on the constraints violations
- MaxIter - maximal number of iterations allowed

2.7 Rounding issues

Input data to the optimization, as well as all intermediate calculations, are not rounded.

The optimized weights that are smaller than *wtol* (i.e. that are essentially zero) are rounded to exact zero.

2.8 Estimation period definition

For the variance estimation procedure we only consider the days when US equity market was actually open.

The same restriction is applied to the liquidity (ADV) estimation.

3 Index Calculation

3.1 Base date

At the Base Date the Index level is equal to 100.

3.2 Trading Days and Holidays

The Index is calculated and disseminated according to the US equity market holidays schedule.

3.3 Calculation Frequency

The Index is calculated in real time every 15 seconds between 9.30 a.m. and 4.00 p.m. Eastern Time (UTC-5). The closing value is calculated at 4:00 p.m. Eastern time.

3.4 Currency

The Index is calculated in USD.

3.5 Index Value Formula

Between two Rebalancing Dates the Index is calculated as follows:

$$Index_t = \frac{\sum_{i=1}^{M} P_t^i q_{Tprev}^i}{D_t}$$

We define a weighting factor of a stock as a ratio of the stock's weight to the stock's price

$$q_t^i = \frac{w_t^i}{P_t^i}$$

At each Rebalancing Date T each stock is assigned a weighting factor that is based on its target weight and the day-T close price. The divisor D transforms the value of the hypothetic index portfolio into index level.

3.6 Treatment of corporate actions and changes in the investment universe

Between two Rebalancing Dates the maintenance of the Index constituents is based on the following principle: all share and price adjustments that do not alter the membership of stocks in the investment universe or their risk characteristics do not lead to changes in the Index value or composition. Below we detail the maintenance rules for the most common corporate

actions. For all the cases not explicitly mentioned in this document the maintenance is made according to the general practices for the S&P index family.

3.6.1 Early exits

If a company that is currently present in the Index is excluded from the investment universe (represented by the S&P 500 Index) between two subsequent Rebalancing Dates, it is immediately excluded from the Index portfolio and its weight is distributed pro-rata among the remaining stocks

$$w_i \to \frac{w_i * 100\%}{1 - w_{exit}}$$

3.6.2 Regular Dividends

Dividends received on a stock present the Index are reinvested in the Index on the net basis. Net dividend is a gross dividend minus withholding tax. Amounts of unadjusted gross cash dividends and withholding tax amounts are calculated using S&P assumptions. The adjustment corresponding to the reinvestment of net dividend is done on the date after the ex-dividend date at the market opening.

Consequently, dividend payment and reinvestment does not change the weights of the Index components, but results in an adjustment of the Index level as follows

$$Index_t \to Index_t(1 + \frac{Div_t}{Index_t})$$

where Div_t is the total net dividend in USD received on the day t and divided by the current index divisor

$$Div_t = \frac{\sum d_i N_i}{D}$$

N_i is a number of shares of the *i*-th company, and d_i is a per-share net dividend expressed in USD. If the company pays dividends in a currency different from USD, the amount of the dividend in converted to USD using the foreign exchange rate at the equity market close.

3.6.3 Special Dividends

Special dividends are those dividends that are outside the normal payment pattern established historically by the issuing corporation. For the detailed definition we refer to the S&P Index Methodology Guide. If a company present in the Index pays a special dividend, the price of the company is adjusted downwards by the amount of the net dividend after the close of trading on the day before the ex-dividend date

$$Adjusted\ Price = Close\ Price - (1 - WTax) * d_i$$

Accordingly, the Index divisor is adjusted to compensate for the price drop

$$D \rightarrow D * \frac{Index - (1 - WTax) * d_i}{Index}$$

This results in a decrease of the company weight in the Index, since its weighting factor remains unchanged.

3.6.4 Spin-offs

If a company present in the index has a spin-off, the spun-off company is not added to the Index. There are two possible scenarios:

1) if the spun-off company has a price, there is a corresponding price drop in the spin-off company, that affects its weight. Then the spun-off company shares are sold and the proceedings are reinvested in the index.
2) If the spun-off company has no price, it is held in the index with a price 0, and there is no adjustment to the price/weight of the spin-off company. When the first trading price for spun-off is available, the scenario 1 is applied.

3.6.5 Merger & Acquisition

We denote companies by the capital letters A, B, C,…

1) Merger: A+B = C
   a. if A and B are in the Index: $w_C = \min(w_A + w_B, w_{max})$
   b. if A is in the Index, and B is not: $w_C = w_A$
2) Acquisition: A+B =A
   a. If A and B are in the Index: $w_A = \min(w_A + w_B, w_{max})$
   b. If only A is in the Index: $w_A = w_A$
   c. If only B is in the Index: $w_B = 0,$

the acquired stock is eliminated from the Index and the proceedings are reinvested pro-rata in the remaining stocks.

The maximal weight limit w_{max} is the same that is used for portfolio optimization constraint.

3.6.6 Share Conversion: A-> B

If a company converts its shares from one class to another, and the class A that was present in the portfolio is converted to a class B that was not, then we keep the converted shares in the Index until the next rebalancing. At the next rebalancing the new share class is considered a new entity, and does not inherit the historical price/volume data of the suppressed share class. This new share class is considered for the inclusion in the Index on the next Rebalancing Date only if it is a component of the investment universe.

3.6.7 Summary Table

EVENT TYPE	IMPACT ON THE INDEX
Company Addition to the investment universe	No change
Company Deletion from the investment universe	If the deleted company is in the Index, it is dropped and its weight is reinvested pro-rata in the remaining stocks
Price Adjustments	Price of the stock and number of shares are adjusted to reflect the corporate action, so the weight of the stock in the index stays the same
Share Issuance/ Buy Back	No change
Rights Offering	Price adjusts down and number of shares adjusts up so the weight of the stock stays the same
Spin-off	The spun-off company is deleted from the Index and its weight is reinvested pro-rata in the remaining stocks
Change of Stock Float Factor	No change
Merger and Acquisition	The acquiring company is given a weight that is a minimum between the sum of the old weights of the acquiring company and the target company in the Index and the maximum weight limit w_{max}. If two companies merge, the weight for the resulting company is a minimum of the sum of the weights of the merging companies and the maximum weight limit w_{max}
Special Dividend	The price of the stock is adjusted down by the amount of the net dividend, with no adjustment to the number of shares, but with the downward adjustment in the Index divisor. The weight of the stock making the special dividend payment goes down
Regular Dividend	The net dividend is reinvested in the total return index only, with

	no adjustment made in the divisor and in the weights

4. Parameters

Tv	50	days liquidity estimation period
q	10	ratio of missing prices accepted in the ADV, Volatility and correlation window
M	250	number of the most liquid stocks
Ts	125 days	volatility estimation period
Tr	500 days	correlation estimation period
Wmax	4.5%	maximum weight
Smax	20%	upper bound for single sector exposure
H	50	inverse diversification target
TolFun	10^{-8}	termination tolerance on the objective function value
TolCon	10^{-8}	tolerance on constraints violation
MaxIter	10^{12}	maximal number of iterations
wtol	10^{-5}	significance threshold for weights
Base Date	2011/05/20	
K	3 days	gap between reference date and Rebalancing Date