Japan Minimum Variance Index

Technical Description

Version 1

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1 INDEX REBALANCING METHODOLOGY

1.1 Review Schedule

For the purpose of this document, a Business Day means any day for which the Japanese equity market is open. The index composition is reviewed monthly. The Rebalancing Date of the index is the third Friday of the month. If the third Friday of the month is not a Business Day, then the Rebalancing Date shifts forward to the first Business Day after the third Friday. The new index composition is effective at the Effective Date (the first Business Day after the Rebalancing Date). The Estimation Date is K Business Days before the Rebalancing Date. The optimal weights of the new index composition are computed the first Business Day after the Estimation Date (the Calculation Date, i.e. K-1 Business Days before the Rebalancing Date) and drifted according to market prices for implementation after the close of Japanese market on the Rebalancing Date.

1.2 Reference Universe

The Investment universe of the Index is the forecasted composition of the S&P Topix 150 index at the Effective Date. Therefore, it is the current (at the Estimation Date) composition of the S&P Topix 150 to which scheduled Additions and Deletions between the Estimation Date and the Effective Date are applied.

1.3 Work Flow

On each Calculation Date, the review process determines a new investable minimum variance index based on the benchmark composition. The process can be summarized by this workflow:

a. Base Universe: Selection from the base index.
b. Liquid Universe: Select the most liquid stocks from the Base Universe.
c. Eligible Universe: Filter out stocks with irregularities in price history from the Liquid Universe.
d. Optimized Portfolio: Weight stocks among the Eligible Universe in order to minimize the total portfolio variance under constraints.
e. Minimum Variance Portfolio: Filter out stocks with negligible weight within the Optimized Portfolio.

1.4 Data

The following data is used in the Index construction process:

- P - daily share prices at market close in JPY,
- TR – daily share total return price. The total return price is adjusted for corporate actions and dividend payments in JPY,
- V - daily transaction volume from the Japan stock exchanges,
- classification of the stocks in S&P 500 by industrial sector corresponding to the level 1 of the GICS classification, having 10 industries:
    1. Energy
    2. Materials
    3. Industrials
    4. Consumer Discretionary
    5. Consumer Staples
    6. Health Care
    7. Financials
    8. Information Technology
    9. Telecommunication Services
    10. Utilities

- Business Date: all trading dates in the Japanese market.

1.5 Determination of the Eligible Universe

1.5.1 Base Universe

The Base Universe is determined as the forward composition of the benchmark index at the Effective Date. The forward composition means the composition adjusted by all reviews (additions/deletions) and corporate actions that will become effective between the current Calculation Date and the current Effective Date.

1.5.2 Liquidity Filter

We design a liquidity filter to restrict on the most liquid stocks of the Base Universe as follows:

a. Liquidity is estimated for each stock, using most recent transaction volume data. Average Daily Volume (ADV) is calculated as a simple average of daily transaction volume series over the past T_v days. Observations with missing volumes are discarded from the mean calculation. If a stock has more than p% missing volume observations during the liquidity estimation period, it is assigned a zero ADV. T denotes estimation date, V is volume in number of shares, P is stock price in the index home currency (JPY).

$$\mathrm{NVD}_T^i = \sum_{t=T-T_v+1}^{T} \mathbb{1}_{V_t^i \text{is missing}}$$

$$\mathrm{ADV}_T^i = \begin{cases} 0 & \text{if } \mathrm{NVD}_T^i > p\% * T_v \\ \dfrac{1}{T_v - \mathrm{NVD}_T^i} \displaystyle\sum_{t=T-T_v+1}^{T} V_t^i * P_t^i * \mathbb{1}_{V_t^i \text{is missing}} & \text{otherwise} \end{cases}$$

b. The stocks from the selection are ranked by their ADV in descending order.
c. The top M stocks having the highest liquidity are selected.

The liquidity filter is applied each time the Index is reviewed, before calculating new optimized weights.

1.5.3 Missing Data Filter: Volatility estimation

The Eligible Universe consists in those stocks whose recent price history does not exhibit large non trading periods, which could bias the volatility estimation for this stock. For this purpose a missing data filter is designed, it performs the following actions:

a. For each stock, count the Number of Non Trading occurrence (NNT) over the past T_s days

$$\mathrm{NNT}_T^i = \frac{1}{T_s} \sum_{t=T-T_s+1}^{T} \mathbb{1}_{A_t^i=\text{'not available'}},\ 1 \leq i \leq M$$

b. Stocks for whose $\mathrm{NNT}_{i >=}$ q% are discarded.

1.5.4 Missing Data Filter: Covariance estimation

The Eligible Universe consists in those stocks whose recent price history does not exhibit large non trading periods, which could bias the covariance estimation for this stock. For this purpose a missing data filter is designed, it performs the following actions:

a. for each stock, count the Number of Non Trading occurrence (NNT) over the past T_r days

$$\mathrm{NNT}_T^i = \frac{1}{T_r} \sum_{t=T-T_r+1}^{T} \mathbb{1}_{A_t^i=\text{'not available'}}$$

b. stocks for whose NNTi >=q% are discarded

1.6 Portfolio Construction

Index constituents are weighted by an optimization procedure which aims to minimize the portfolio variance under constraints.

1.6.1 Return Data

The optimization procedure starts by calculating daily arithmetic price returns:

$$r_t^i = \frac{TR_t^i}{TR_{t-1}^i} - 1$$

where t-1 denotes the previous business day, TR are total return close prices.

1.6.2 Variance Estimation Details

For all the stocks admitted to the optimization step a variance-covariance matrix is estimated as follows:

$$\Sigma_{\mathrm{T}}^{\mathrm{i,j}} = \sigma_{\mathrm{T}}^{\mathrm{i}} * \sigma_{\mathrm{T}}^{\mathrm{j}} * \rho_{\mathrm{T}}^{\mathrm{ij}}$$

where M is the number of stocks admitted for optimization, σ^i is volatility of the i-th stock, and ρ^{ij} is correlation between the stocks (*i,j*). The ingredients of the covariance matrix are estimated on arithmetic daily returns as follows:

$$\sigma_{\mathrm{T}}^{\mathrm{i}} = \sqrt{\frac{1}{T_s - 1} \sum_{\mathrm{t=T-}T_s+1}^{\mathrm{T}} (\mathrm{r}_{\mathrm{t}}^{\mathrm{i}} - \bar{\mathrm{r}}^{\mathrm{i}})^2}$$

where T denotes Estimation date and Ts is volatility estimation period in days, $\bar{r}$ denotes simple average of stock returns. And correlation coefficients are estimated as

$$\rho_{\mathrm{T}}^{\mathrm{i,j}} = \frac{\frac{1}{\mathrm{T}_r - 1} \sum_{\mathrm{t=T-T}_r+1}^{\mathrm{T}} (\mathrm{r}_{\mathrm{t}}^{\mathrm{i}} - \bar{\mathrm{r}}^{\mathrm{i}})(\mathrm{r}_{\mathrm{t}}^{\mathrm{j}} - \bar{\mathrm{r}}^{\mathrm{j}})}{\sigma_{\mathrm{T}}^{\mathrm{i}} * \sigma_{\mathrm{T}}^{\mathrm{j}}}$$

where T denotes Estimation date and Tr is correlation estimation period in days, and volatilities in the denominator are estimated over the Tr-day period. Only the days when all the stocks were actually traded are included in the estimation of the means and the variance-covariance

1.6.3 Optimization: objective function

The function to be minimized is the variance of the Index portfolio:

$$\sigma_{Ind}^2 = \sum_{i=1}^{M} \sum_{j=1}^{M} w_i \Sigma_{\mathrm{ij}} \mathrm{w_j}$$

1.6.4 Optimization: constraints

The optimization is subject to the following constraints:

- 100% leverage constraint: $\sum_{i=1}^{M} w_i = 1$
- long-only constraint: $w_i \geq 0$, for all i
- maximal weight constraint: $w_i \leq w_{max}$
- maximal sector exposure constraint: $w_S \leq S_{max}$ where $w_S = \sum_{i \in S} w_i$, is net exposure to the sector S.
- diversification target: $\sum_{i=1}^{M} w_i^2 = \frac{1}{H}$

1.6.5 Optimization: numerical algorithm

The optimization problem is a quadratic constrained minimization problem. It is solved numerically, using an interior-point algorithm. This algorithm calculates an iterative sequence of approximate minimization problems, where inequality constraints are transformed into equality constraints using slack variables. The optimal solution is defined with the help of the following convergence criteria:

- TolFun - termination tolerance on the function value
- TolCon - tolerance on the constraints violations
- MaxIter - maximal number of iterations allowed

1.7 Determination of the Minimum Variance Portfolio

As Input data to the optimization as well as intermediate calculations are not rounded, weights in the Optimized Portfolio may be arbitrarily small. Components whose weights are negligible are rounded to 0 and effectively removed from the Minimum Variance Portfolio. A threshold of w_{tol} is used to determine if a weight is negligible. The total weight from negligible stocks that is rounded off is redistributed pro-rata to the remaining stocks.

$$w_i \rightarrow \begin{cases} 0 & \text{if } w_i < w_{tol} \\ \frac{w_i}{\sum_j w_j * \mathbb{1}_{w_j \geq w_{tol}}} & \text{otherwise} \end{cases}$$

1.8 Parameters

- p = 10% maximum share of missing values inside liquidity estimation period accepted
- Tv = 50 days liquidity estimation period
- M = 140 number of the most liquid stocks selected by liquidity filter
- Ts = 125 days variance estimation period
- Tr = 500 days covariance estimation period
- q = 10% maximal share of missing values inside volatility/covariance estimation period
- W_{max} = 4.5% maximal weight
- S_{max} = 20% upper bound for single sector exposure
- Sect. Classif = GICS Sector classification
- H = 50 inverse diversification target
- TolFun = 10^{-8} termination tolerance on the objective function value
- TolCon = 10^{-8} tolerance on constraints violation
- MaxIter = 10^{12} maximal number of iterations
- wtol = 10^{-5} significance threshold for weights
- K = 4 days gap between Estimation date and Rebalancing Date

2 TREATMENT OF CORPORATE ACTIONS AND CHANGES IN THE INVESTMENT UNIVERSE

Between two Rebalancing Dates the maintenance of the Index constituents are based on the following principle: all share and price adjustments that do not alter the membership of stocks in the investment universe or their risk characteristics do not lead to changes in the Index value or composition. Below details the maintenance rules for the most common corporate actions. For all the cases not explicitly mentioned in this document the maintenance is made according to S&P corporate action methodology.

2.1 Early exits

If a company that is currently present in the Index is excluded from the investment universe (represented by the S&P Topix 150 Index) between two subsequent rebalancing dates, it is immediately excluded from the Index portfolio and its weight is distributed pro-rata among the remaining stocks

$$w_i \rightarrow \frac{w_i * 100\%}{1 - w_{exit}}$$

2.2 Spin – offs

If a company present in the index has a spin-off, the spun-off company is not added to the Index. There are two possible scenarios:

2.2.1 If the spun-off company has a price, there is a corresponding price drop in the spin-off company that affects its weight. Then the spun-off company shares are sold and the proceedings are reinvested in the index.

2.2.2 If the spun-off company has no price, it is held in the index with a price 0, and there is no adjustment to the price/weight of the spin-off company. When the first trading price for spun-off is available, the scenario 1 is applied.

2.3 Merger & Acquisition

We denote companies by the capital letters A, B and C.

1) Merger: A+B = C
    a. if A and B are in the Index: $w_C = \min(w_A + w_B, w_{max})$
    b. if A is in the Index, and B is not: $w_C = w_A$
2) Acquisition: A+B =A
    a. If A and B are in the Index: $w_A = \min(w_A + w_B, w_{max})$
    b. If only A is in the Index: $w_A = w_A$
    c. If only B is in the Index: $w_B = 0$, the acquired stock is eliminated from the Index and the proceedings are reinvested pro-rata in the remaining stocks.

The maximal weight limit w_{max} is the same that is used for portfolio optimization constraint.

2.4 Share Conversion: A-> B

If a company converts its shares from one class to another, and the class A that was present in the portfolio is converted to a class B that was not, then we keep the converted shares in the Index until the next rebalancing. At the next rebalancing the new share class is considered a new entity, and does not inherit the historical price/volume data of the suppressed share class. This new share class is considered for the inclusion in the Index on the next rebalancing date only if it is a component of the investment universe.

2.5 Summary Table

EVENT TYPE	IMPACT ON THE INDEX
Company Addition to the investment universe	No change
Company Deletion from the investment universe	If the deleted company is in the Index, it is dropped and its weight is reinvested pro-rata in the remaining stocks
Price Adjustments	Price of the stock and number of shares are adjusted to reflect the corporate action, so the weight of the stock in the index stays the same
Share Issuance/ Buy Back	No change
Rights Offering	Price adjusts down and number of shares adjusts up so the weight of the stock stays the same
Spin-off	The spun-off company is deleted from the Index and its weight is reinvested pro-rata in the remaining stocks
Change of Stock Float Factor	No change
Merger and Acquisition	The acquiring company is given a weight that is a minimum between the sum of the old weights of the acquiring company and the target company in the Index and the maximum weight limit w_{max}. If two companies merge, the weight for the resulting company is a minimum of the sum of the weights of the merging companies and the maximum weight limit w_{max}
Special Dividend	The price of the stock is adjusted down by the amount of the net dividend, with no adjustment to the number of shares, but with the downward adjustment in the Index divisor. The weight of the stock making the special dividend payment goes down
Regular Dividend	The net dividend is reinvested in the total return index only, with no adjustment made in the divisor and in the weights

3 HEDGED VERSION OF THE JAPAN MINIMUM VARIANCE INDEX

There exist a Euro-Hedged version of the Japan Minimum Variance Index. The hedging is performed monthly, according to the S&P general methodology available at http://eu.spindices.com/documents/methodologies/methodology-index-math.pdf?force_download=true